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Section 1: Introduction to Geometry
Student Learning Plan

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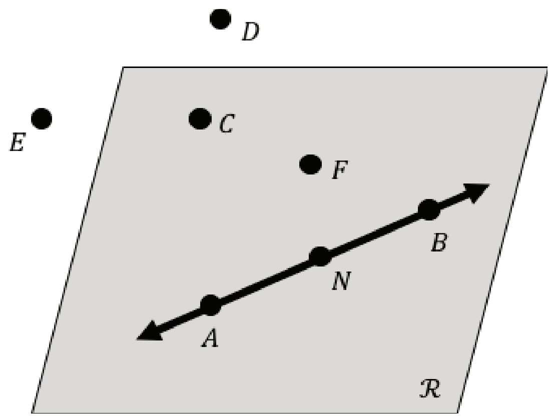
What did you learn in this section? What questions do you still have?
Who was your favorite Study Expert for this section? Why?

Section 1 – Topic 1
Basics of Geometry – Part 1

1. Consider the diagram below.



Identify whether or not the following features are represented in the above diagram. For each feature that is represented, write the name(s) denoted in the diagram.

Feature	Name(s) denoted in diagram
Ray	
Vertex	
Angle	
Parallel lines	
Parallel planes	
Coplanar points	
Collinear points	
Segment Addition Postulate	
Perpendicular lines	

2. Mrs. Fiorina wrote the following statement on the board.

"A ray can be part of a line."

A group of students was discussing the meaning of this statement. They were arguing back and forth about whether or not the converse of the statement was also true. Jahmiah, the leader of the group, determined that the converse was not true.

Is Jahmiah correct or not? Justify your answer.

3. Make a list of complete sentences that describe five applications of the building blocks of geometry in the real world.

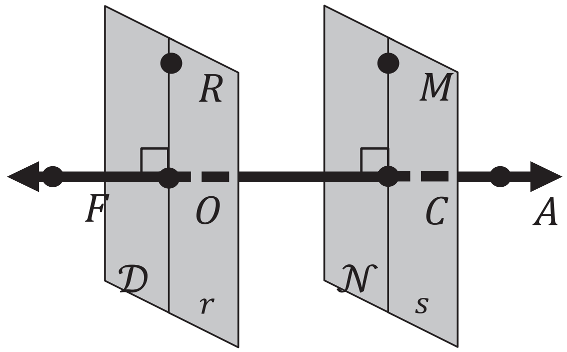
4. Consider the plane, $\mathcal{T}$, and three collinear (but not coplanar) points, P, Q, and W.

Part A: Can either P, Q, or W be a point on $\mathcal{T}$? Justify your answer.

Part B: If the lengths of $\overline{PW}$ and $\overline{PQ}$ are given, can the length of $\overline{QW}$ be determined? Justify your answer.

Section 1 – Topic 2
Basics of Geometry – Part 2

1. Consider the diagram below with parallel planes $\mathcal{D}$ and $\mathcal{N}$.



For each statement written below, determine whether the statement is true or false.

Statement	True or False?
FA is a line, line segment, and a ray.	O True O False
Lines r and s are perpendicular.	O True O False
$\overrightarrow{RO}$ and $\overrightarrow{CA}$ form an angle.	O True O False
M and C are collinear and coplanar.	O True O False
F, O, and R are coplanar in $\mathcal{D}$.	O True O False
$\overrightarrow{MC}$ and $\overrightarrow{CA}$ form a $90°$ angle.	O True O False
Using the Segment Addition Postulate, $FO + OC = FA$.	O True O False
$\overleftrightarrow{AF}$ intersects both $\mathcal{D}$ and $\mathcal{N}$ perpendicularly.	O True O False

2. Complete the following statements with a word or phrase and draw a representation under each statement.

➢ Through any _____________ non-collinear points there is exactly one plane.

➢ If two points lie in a plane, then ______________________________________

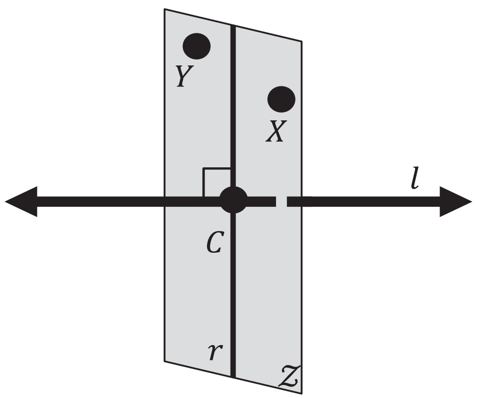
____.

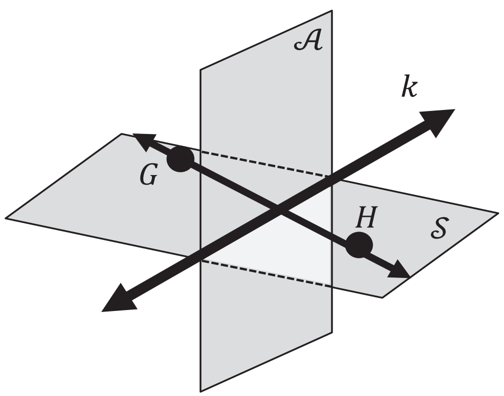
➢ If two planes intersect, they intersect in exactly one _____________.

➢ Given ____, there is one, and only one, line perpendicular to the plane through that point.

3. Sketch a plane containing points E, D, and B; where E and B are collinear with A, and D is a vertex where $\overrightarrow{CD}$ and $\overrightarrow{FD}$ meet.

4. Compare and contrast the following diagrams by creating a list of at least five complete sentences.





Section 1 – Topic 3
Introduction to Proofs

1. Define the following terms.

Conjecture:

Inductive Reasoning:

Deductive Reasoning:

Counterexample:

2. Mr. Epstein placed two-stools in front of the classroom. One had three-legs and the other had four legs. He asked Charlene to sit on each of them. She noticed that the four-legged stool rocked when she sat on it, but the three-legged stool was steady and did not rock.

   Make a conjecture about why this might have occurred.

3. Provide a counterexample to show that the statement is false. You may use words or draw a diagram.

If $\overline{AB} \cong \overline{BC}$, then B is the midpoint of $\overline{AC}$.

4. Which type of reasoning is being used in each of the following situations? State how you know.

It has rained the past 3 days. Joaquin makes a conjecture that it will rain again tomorrow.

Inductive Deductive

Maria is writing a list of numbers. She asked Susanne what the next number would be. After looking at the list: 7, 14, 21, 28, 35, 42, _____ Susanne states, "the next number will be 49."

Inductive Deductive

Ky adds the sides of an equilateral triangle and finds the length of the perimeter to be 3 times the length of one side. He guesses that the perimeter of every equilateral triangle will be three times the length of a side.

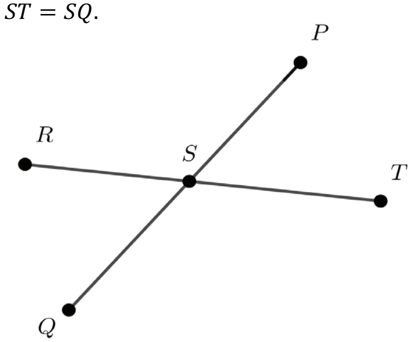
Inductive Deductive

5. Use the diagram to complete the proof by filling in the blanks.

Given: $\overline{RT}$ and $\overline{PQ}$ intersect at point S so that

$RS = PS$ and $ST = SQ$.

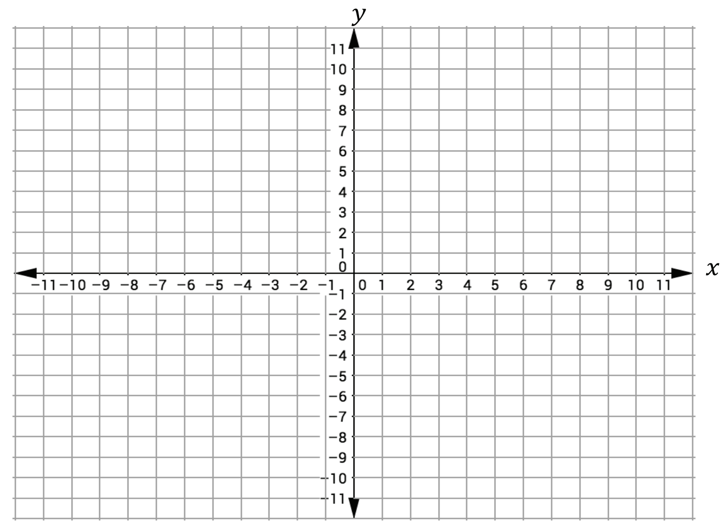
Prove: $RT = PQ$



Statements	**Reasons**
1. Given	1. Given
2. $RS + ST = PS + SQ$	2. Addition Property
3. $RS + ST = RT;\ PS + SQ = PQ$	3.
4.	4. Substitution Property

Section 1 – Topic 4
Midpoint and Distance in the Coordinate Plane – Part 1

1. Consider the following coordinate plane.

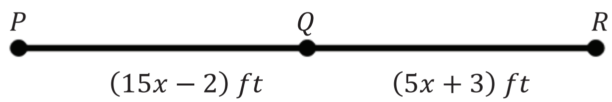


Part A: Plot the points $A(-8, 7)$ and $B(6, -9)$. Mark the halfway point on $\overline{AB}$ and label it point M. What are the coordinates of M?

Part B: Plot point $C(2, 7)$. If M is the midpoint of $\overline{CD}$, what are the coordinates of D?

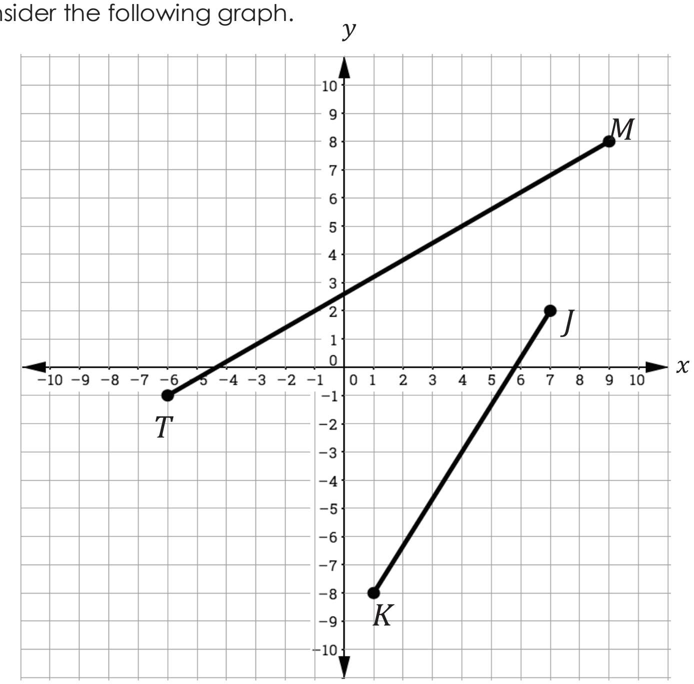
2. Given $H(6,7)$ and $I(-7,-6)$, if point G lies $\frac{1}{2}$ of the way along $\overline{HI}$, Santiago argues that point G is located at the origin. Is Santiago correct? Justify your answer.

3. Consider the line segment below that is five feet long. Is Q the midpoint of $\overline{PR}$? Justify your answer.



4. Rihanna works in a coffee shop approximately nine miles from her apartment. She bikes every day from her apartment to the coffee shop and then back to her apartment in the evening. On her way to work, she always stops halfway through to meet her best friend at a park. If the distance from Rihanna's apartment to the park is $(5x+2)$ miles and the trip from the park to the coffee shop is $(25x-8)$ miles long, then what is the value of x?

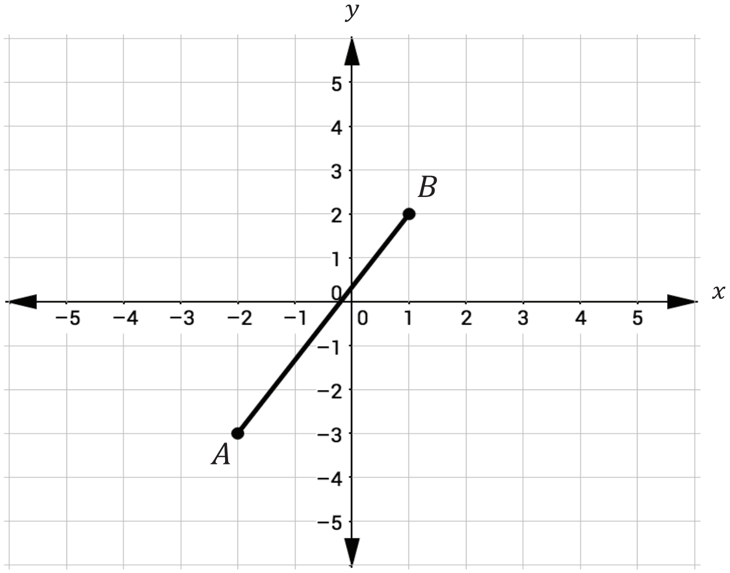
5. Consider the following graph.



If P and Q are the midpoints of $\overline{MT}$ and $\overline{JK}$, respectively, then sketch $\overline{PQ}$ in the above coordinate plane.

6. M is the midpoint of $\overline{GR}$. G has coordinates $(-8,3)$ and M is at the origin. Find the coordinates of R.

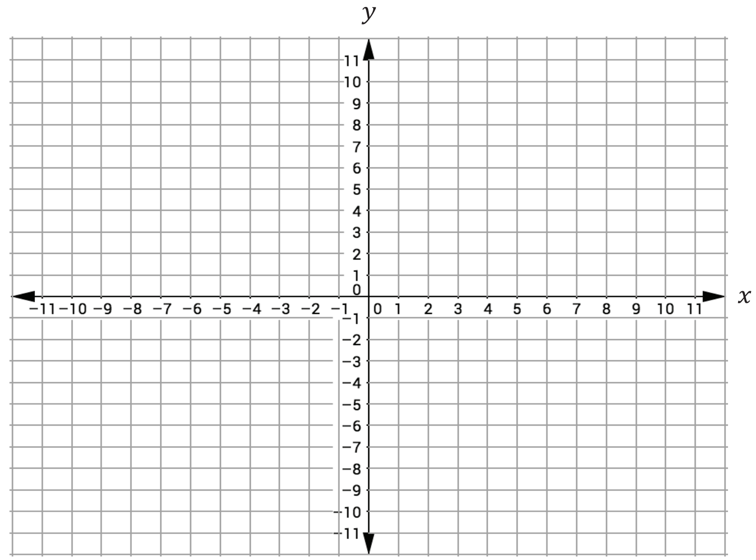
7. Consider the following graph.



If $\overline{BA}$ is extended all the way through A creating $\overline{BF}$ and A becomes the midpoint of $\overline{BF}$, then what are the coordinates of F?

Section 1 – Topic 5
Midpoint and Distance in the Coordinate Plane – Part 2

1. Consider the following coordinate plane.



Part A: Plot the points $D(-6,-2)$ and $T(7,-8)$. Describe how to find the distance between D and T using the Pythagorean Theorem.

Part B: Determine the length of $\overline{DT}$. Round your answer to two decimal places.

2. Two horses are ready to return to their barn after a long workout session at the track. The horses are at coordinates $H(1, 10)$ and $Z(10, 1)$. Their barns are located in the same building, which is at coordinates $B(-3, -9)$. Each unit/grid on the coordinate plane represents 100 meters. Which horse is closer to the barn? Justify your answer.

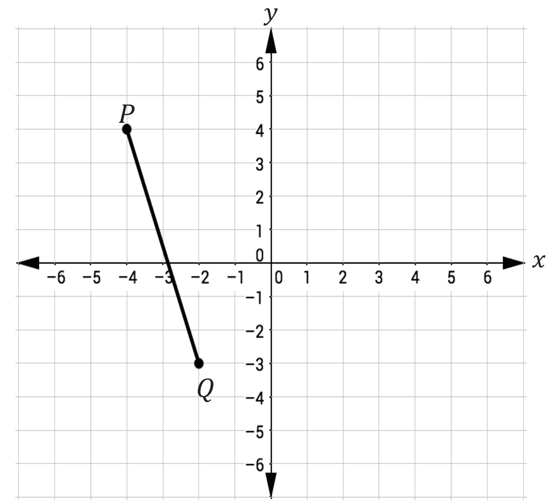
3. Darko found the distance between points $A(3, -4)$ and $B(-1, 3)$. His work is shown below.

$x_A = 3 \quad y_A = -4 \qquad x_B = -1 \quad y_B = 3$

$$D(A,B) = \sqrt{(-1-3)^2 + (3-4)^2}$$
$$= \sqrt{(-4)^2 + (-1)^2}$$
$$= \sqrt{16 + 1}$$
$$= \sqrt{17}$$
$$\approx 4.123$$

He wants you to check his work before giving it to his teacher. Determine if Darko's work is correct. Explain each of the steps that he followed and identify and correct any errors, if necessary. Round your answer to two decimal places.

4. Consider the following graph.



Part A: Plot two points to form a square in the above graph. Label the points R and S. A square is a quadrilateral with all four sides of the same length. After plotting the points, trace the square by connecting the points.

Part B: To find the perimeter of a polygon, take the sum of the length of each side. What is the perimeter of the square that you created in *Part A*? Round your answer to two decimal places.

5. Consider a triangle with vertices at $S(-2,-3)$, $A(2,3)$, and $N(5,-4)$.

   Part A: What is the shortest side of the triangle? Select the correct response.

   Ⓐ $\overline{SA}$
   Ⓑ $\overline{AN}$
   Ⓒ $\overline{NS}$
   Ⓓ All sides are congruent.

   Part B: Justify your answer from *Part A*.

Section 1 – Topic 6
Partitioning a Line Segment – Part 1

1. What is the value of k used to find the coordinates of a point that partitions a segment into a ratio of $5:3$?

2. Determine the value of k that partitions a segment into a ratio of $1:4$.

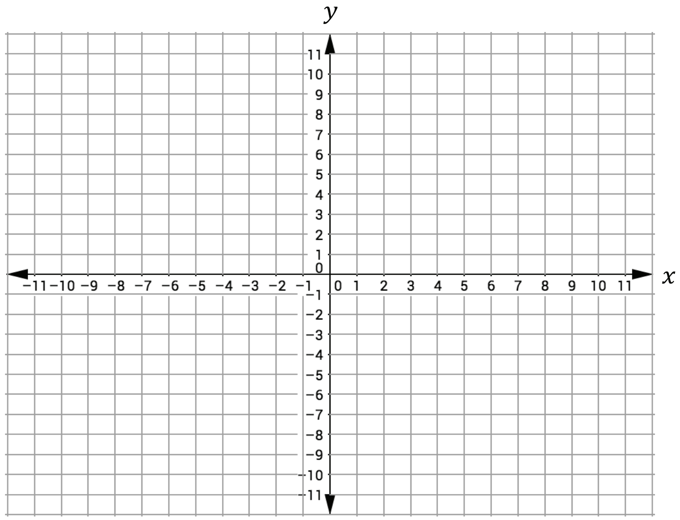
3. Given $G(1,2)$ and $K(8,12)$.

   Part A: What does it mean to find the point P on $\overline{GK}$ such that $3(GP) = 2(PK)$?

   Part B: What are the coordinates of P?

4. Given the points $M(-3,-4)$ and $T(5,0)$, find the coordinates of the point Q on directed line segment $\overline{MT}$ that partitions $\overline{MT}$ in the ratio $2:3$.

5. Point A has coordinates $(7,2)$. Point B has coordinates $(2,-10)$. Find the coordinates of point P that partition $\overline{AB}$ in the ratio $5:1$.



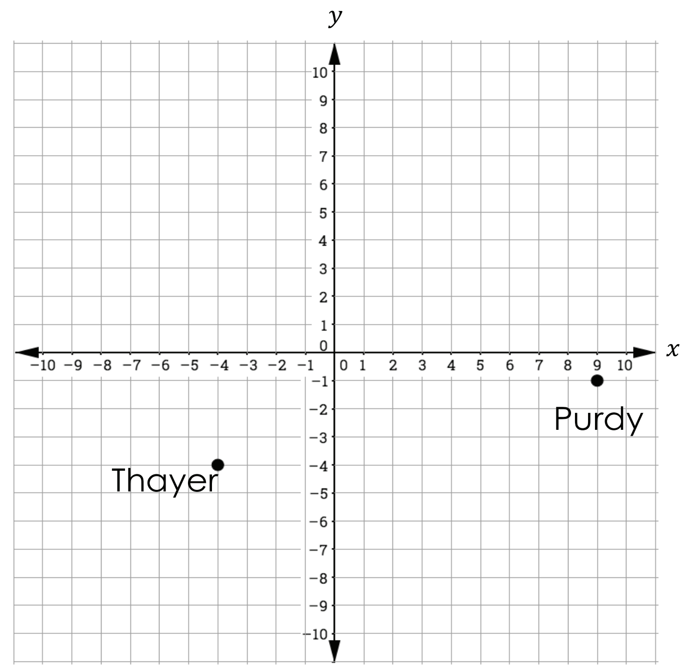
6. $\overline{AB}$ has coordinates $A(-5,9)$ and $B(7,-7)$. Points P, Q, and T are collinear points in $\overline{AB}$ with coordinates $P(-2,5)$, $Q(1,1)$, and $T(4,-3)$.

Part A: Which of the following line segments would contain the point that partitions $\overline{AB}$ into a ratio of $3:2$?

Ⓐ $\overline{AP}$
Ⓑ $\overline{PQ}$
Ⓒ $\overline{QT}$
Ⓓ $\overline{TB}$

Part B: Justify your answer from *Part A*.

7. The following map shows two newly developed towns.



The planners in the local department of transportation want to build a highway with two rest stops between the towns. The rest stops will divide the highway into three equal parts.

What are the coordinates of the points at which the rest stops should be built?

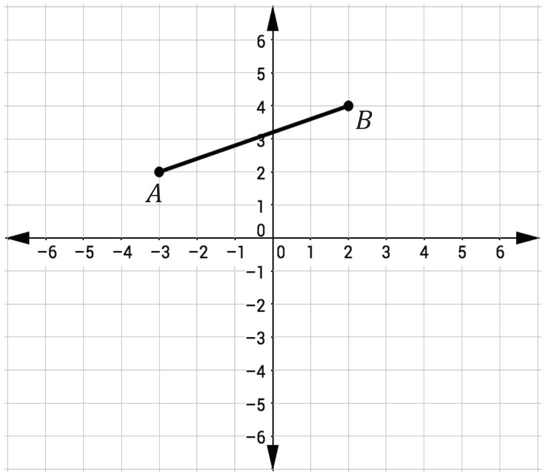
8. Points A, B, and C are collinear on $\overline{AC}$, and $AB{:}BC = \frac{3}{4}$. A is located at (x, y), B is located at $(4, 1)$, and C is located at $(12, 5)$. What are the values of x and y?

Section 1 – Topic 7
Partitioning a Line Segment – Part 2

1. What is the difference between the ratio $AP{:}\,PB$ and the ratio of $AP{:}\,AB$?

2. Points G, K, and J are collinear on $\overline{GJ}$, and $GK{:}GJ = \frac{3}{5}$. G is located at $(-4, 5)$, K is located at (x, y), and J is located at $(6, 0)$. What are the values of x and y?

3. Consider the line segment in the graph below.



Suppose R is plotted in the above coordinate plane and is collinear with A and B. If the ratio $AR : AB$ is $\frac{1}{3}$, then what are the coordinates of R?

4. Three collinear points on the coordinate plane are

$$R(x, y)$$
$$S(x + 8h, y + 8k)$$
$$P(x + 6h, y + 6k).$$

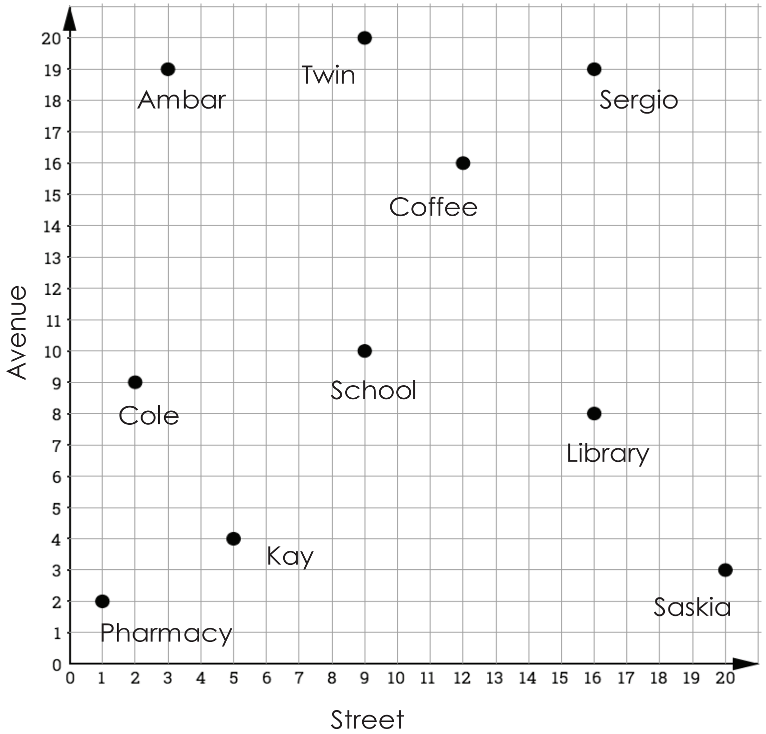
Part A: Determine the value of $\frac{RP}{SP}$.

Part B: Determine the value of $\frac{RP}{RS}$.

5. Points A, P, and B are collinear on $\overline{AB}$, and $PB : AB = \frac{2}{7}$. A is located at the origin, P is located at $(10, -5)$, and B is located at (x, y). Which axis is closer to B, the x-axis or the y-axis? Justify your answer.

6. Points M, T, and D are collinear on $\overline{MD}$. M is located at $(-3, -5)$, T is located at $(1, -3)$, and D is located at $(9, 1)$. What is the value for the ratio $MT : MD$?

7. Use the map and the information given to solve each problem that follows.



Part A: Consider the distance from the school to the coffee shop and the distance from the library to the coffee shop. Draw their respective line segments in the above graph. If the school is relocated in a way that it is collinear to the coffee shop and the library, and the distance to the coffee shop is preserved, then the point representing the school will partition the line segment from the coffee shop to the library in a $x: y$ ratio. What are the values of x and y?

Part B: Sergio lives at the corner of 16^{th} Street and 19^{th} Avenue. Saskia lives at the corner of 20^{th} Street and 3^{rd} Avenue. Located $\frac{1}{4}$ of the distance from Sergio's place to Saskia's place is the post office. Where is the post office located?

Part C: Kay lives is at the corner of 5^{th} Street and 4^{th} Avenue. Her boyfriend works at the Twin Theater located at the corner of 9^{th} Street and 20^{th} Avenue. They have lunch every Tuesday and Thursday at a market located $\frac{3}{4}$ of the distance from the Twin Theater and Kay's house. Where is the market located?

Part D: Ambar and Cole are engaged. Ambar works at the pharmacy and Cole works at the Library. In a few months, they will move to an apartment that is located at a point that is collinear to the locations of the pharmacy and the library and partitions the line segment into a ratio of $2: 3$ coming from the pharmacy. If each grid in the graph represents 500 meters, then how far is the apartment from both Cole's and Ambar's places of work?

Section 1 – Topic 8
Parallel and Perpendicular Lines – Part 1

1. Indicate whether the lines are parallel, perpendicular, or neither. Justify your answer.

   a. $y = 4x - 1$ and $12x = 3y + 7$

   b. $x - 7y = 10$ and $2x + 14y = 21$

   c. $5x + 6y = 18$ and $18x - 15y = 36$

   d. $x = -1$ and $y = -1$

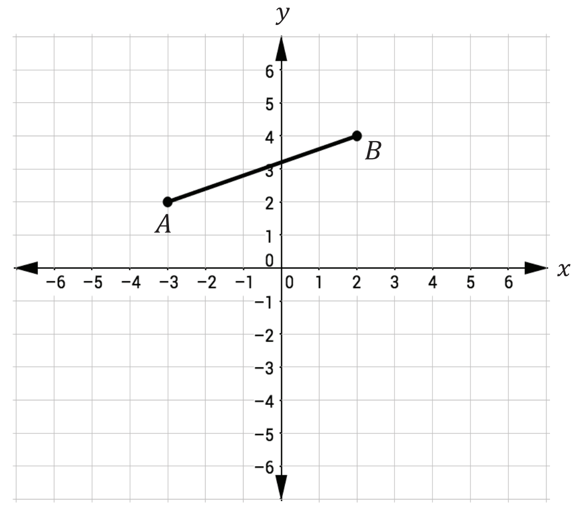
2. Write the equation of the line that it is parallel to $2x = 1 - 3y$ and passes through $(9, 4)$.

3. Write the equation of the line that it is perpendicular to $5x + 8y = 16$ and passes through $(-5, 7)$.

4. Match each of the following with the equations below. Write the letter of the appropriate equation in the column beside each item.

_______	A line parallel to $y = \frac{1}{3}x + 2$	**A.** $y = 0$
_______	A line perpendicular to $x = 3$	**B.** $y = -\frac{1}{3}x + 1$
_______	A line perpendicular to $9x - 3y = 18$	**C.** $x = 3y + 21$
_______	A line parallel to $-4x + 8y = 9$	**D.** $x - 2y = -2$

5. Consider the line segment in the graph below.



Part A: Draw a line segment parallel to $\overline{AB}$ passing through $(-4, -1)$, that has the same length of $\overline{AB}$. Name the line segment $\overline{MY}$.

Part B: Draw a line segment perpendicular to $\overline{AB}$ passing through $(-3, 4)$ and intersecting both $\overline{AB}$ and $\overline{MY}$. Name the line segment $\overline{QU}$.

Section 1 – Topic 9
Parallel and Perpendicular Lines – Part 2

1. Write the equation of the line that it is perpendicular to $y = 7x - 3$ and passes through the origin.

2. Write the equation of the line that it is parallel to $y = 2x + 1$ and passes through the solution of the following system of equations.

$$\begin{cases} 3x - 2y = 10 \\ x + y = 5 \end{cases}$$

3. The equation for line A is given by $y = \frac{1}{3}x + 4$. Suppose line A is parallel to line B and line T is perpendicular to line A. Point $(-3, 1)$ lies on both line B and line T.

   Part A: Write an equation for line B.

   Part B: Write an equation for line T.

4. A rectangle is a four-sided flat shape where every interior angle is a right angle. Therefore, opposite sides are parallel and consecutive sides are perpendicular. Rae is drawing rectangle $PQRS$ on a coordinate plane. The rectangle has coordinates $P(-1, 2)$, $Q(2,4)$, $R(x, y)$, and $S(3, -4)$.

   Part A: What are the values of x and y?

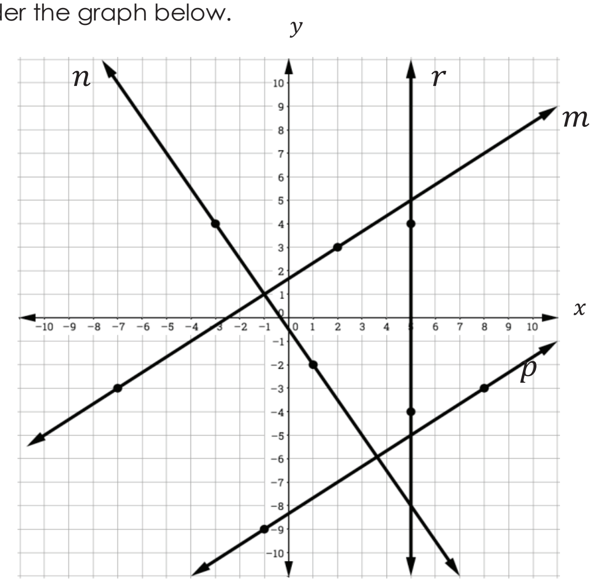
   Part B: Determine if $\overline{PQ}$ and $\overline{RS}$ are parallel.

   Part C: Determine if $\overline{QR}$ and $\overline{SP}$ are parallel.

   Part D: Determine if $\overline{PQ}$ and $\overline{QR}$ are perpendicular.

   Part E: Describe the relationship between $\overline{SP}$ and $\overline{RS}$.

5. Consider the graph below.

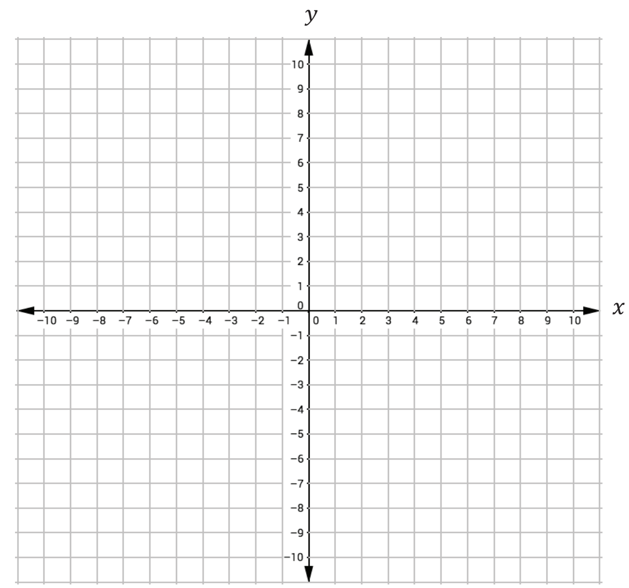


Part A: Name a set of lines that are parallel. Justify your answer.

Part B: Name a set of lines that are perpendicular. Justify your answer.

Section 1 – Topic 10
Introduction to Coordinate Geometry

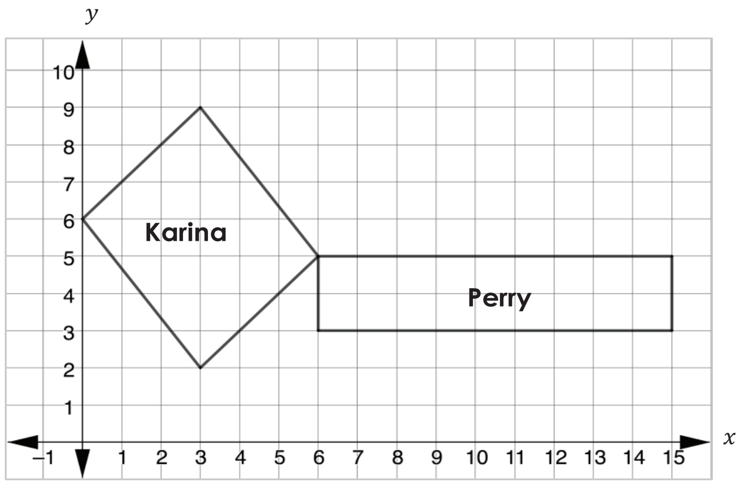
1. Given $R\,(7,-1), A(3,-6), B(-3,-6), E(-5,4)$, plot the points and trace the figure.



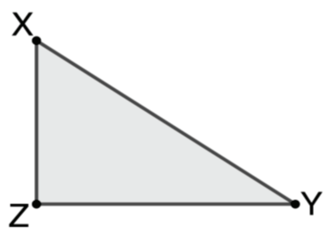
Part A: Determine the lengths of each side (round to the nearest hundredth).

Part B: Determine the perimeter.

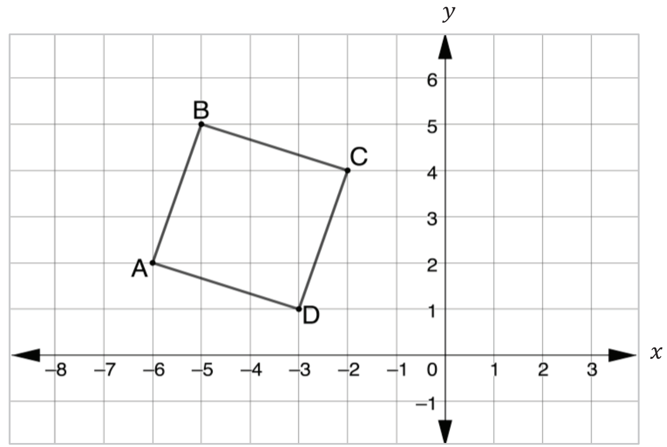
2. Karina and Perry are cleaning up litter in the park for community service hours. Karina and Perry both claim they have covered the greatest area. Who is correct? Justify your answer using mathematical reasoning.



3. ΔXYZ with vertices $X(1,5)$ and $Z(1,1)$ has an area of 10 units². What are the coordinates of the third vertex?



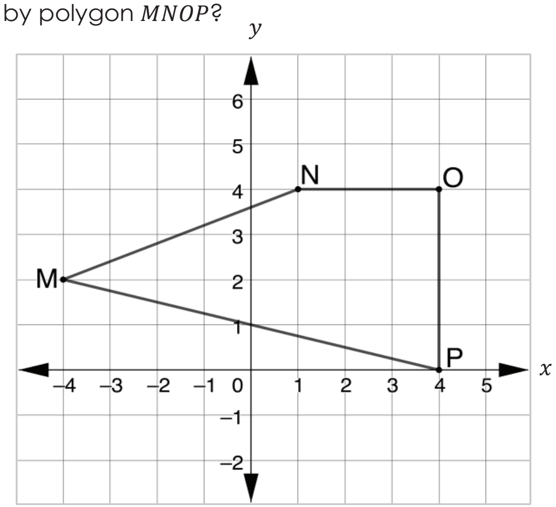
4. Consider the diagram of quadrilateral $ABCD$.



Part A: Prove that $\overline{AB} \perp \overline{AD}$.

Part B: Prove that $\overline{AD} \parallel \overline{BC}$.

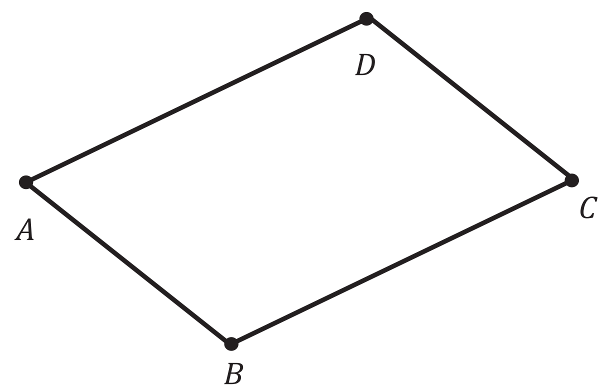
5. The local hardware store sells fencing for $\$1.30$ per yard. If each unit on the grid represents 25 yards, how much will it cost to fence in the plot of land represented by polygon $MNOP$?



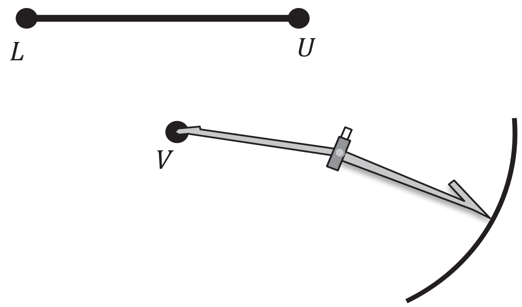
Section 1 – Topic 11
Basic Constructions – Part 1

1. Copy $\overline{HT}$ and construct it in the space provided below.

2. Construct $\overline{GR}$, a copy of $\overline{AB}$.

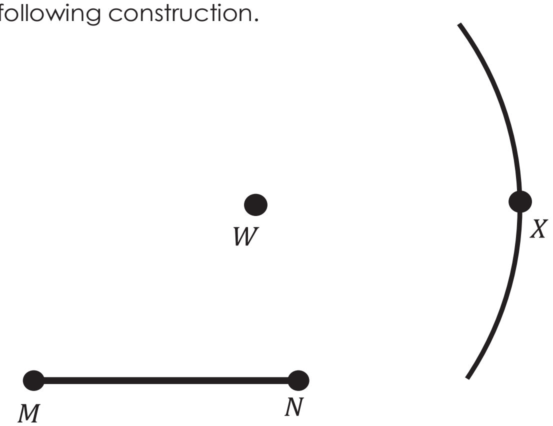


3. Consider the following construction.



Determine the type of construction shown above and describe the next steps to complete the construction.

4. Consider the following construction.



How can you justify that the length of $\overline{WX}$ is equivalent to the length of $\overline{MN}$?

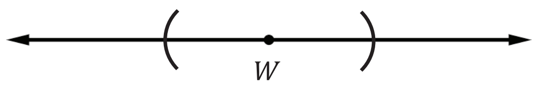
5. The following paragraph describes the process of copying $\overline{TV}$. Strikethrough the words or sentences that make this paragraph wrong and rewrite them correctly in the space provided below.

Mark a point P that will be one endpoint of the new line segment. Set the point of the compass on point T of the line segment to be copied. Adjust the width of the compass to about halfway of point V. The width of the compass is now equal to the length of $\overline{TV}$. Extend the width of the compass about a quarter, place the compass point on P. Keeping the same width on the compass, draw an arc approximately where the other endpoint will be created. Pick a point N on one of the endpoints of the arc and make that the other endpoint for the new line segment. Use the straightedge to draw a line segment from P to the point on the arc.

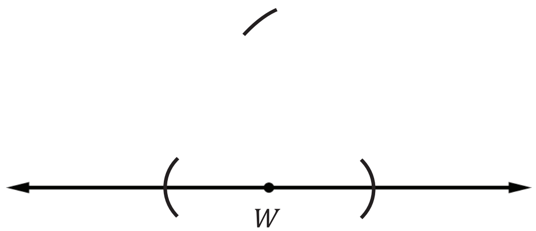
Section 1 – Topic 12
Basic Constructions – Part 2

1. Determine the type of construction from the following steps.

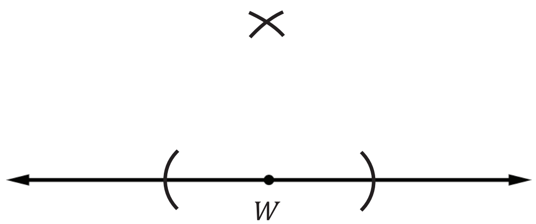
Step 1:



Step 2:



Step 3:



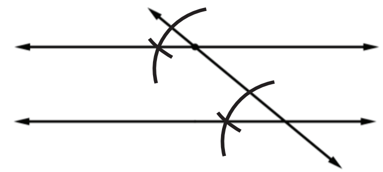
Ⓐ Parallel
Ⓑ Perpendicular
Ⓒ Congruent
Ⓓ Similar

2. When constructing a line parallel to a given line, you will be:

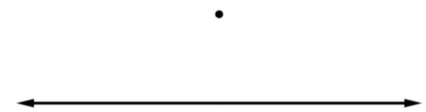
Ⓐ Copying a segment.
Ⓑ Bisecting a segment.
Ⓒ Copying an angle.
Ⓓ Constructing a perpendicular.

3. The diagrams below model the steps for a parallel line construction.

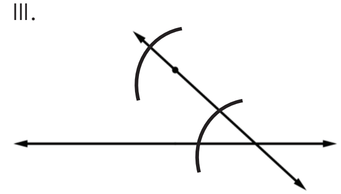
I.



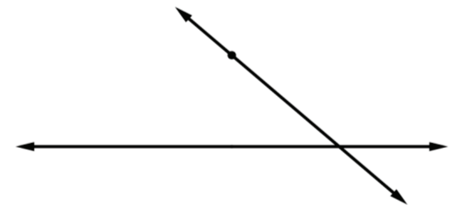
II.



III.



IV.



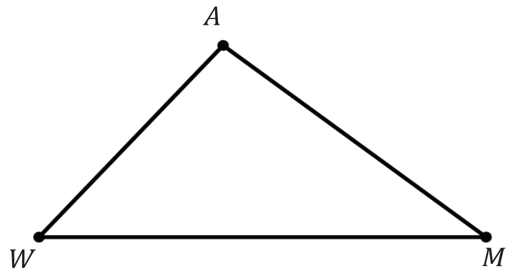
List the constructions in the correct order.

4. Describe the differences between the first step of constructing a parallel line and a perpendicular line.

5. Describe the differences, if any, of constructing a perpendicular line from a point that is ON the line and NOT ON the line.

Section 1 – Topic 13
Constructing Perpendicular Bisectors

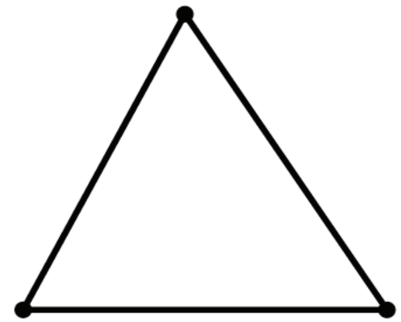
1. Construct a perpendicular bisector of $\overline{WA}$ in ΔWAM below.



2. Label the steps to construct a perpendicular bisector in the correct order below for line segment $\overline{MO}$.

______	Draw large arcs both above and below the middle of $\overline{MO}$.
______	Start with $\overline{MO}$.
______	With your straightedge, connect the two points of where the arcs intersect.
______	Without changing the width of the compass, place the compass on point O and draw two arcs so that they intersect the arcs previously drawn.
______	Place your compass point on M, and stretch the compass more than halfway to point O.

3. Construct or sketch the three perpendicular bisectors of the triangle below.



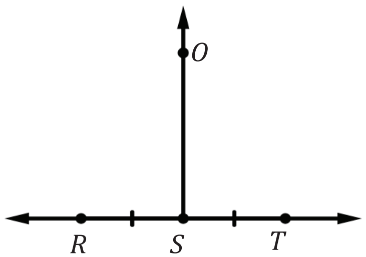
4. Write the term that best identifies the given definitions.

______________	This is a line, segment, or ray that passes through another segment and cuts that segment into two congruent parts.
______________	This is an educated guess based on what you know or observe.
______________	This divides lines, angles, and shapes into two equal parts.
______________	This is a segment that cuts a segment in half and is also $90°$ to the segment that it cuts in half.

Section 1 – Topic 14
Proving the Perpendicular Bisector Theorem Using Constructions

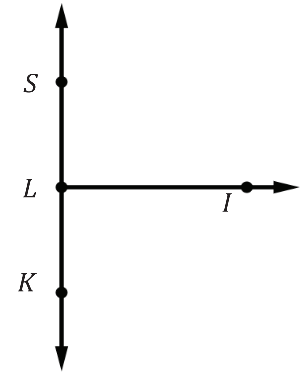
1. Define the perpendicular bisector theorem.

2. Consider the figure below where $\overline{OS}$ is perpendicular to $\overline{RT}$.



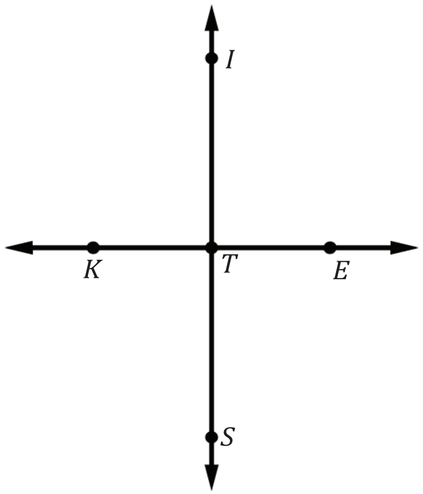
If $\overline{RS} \cong \overline{ST}$ and $RO = 13$, then determine the $m\overline{OT}$.

3. Consider the figure below.



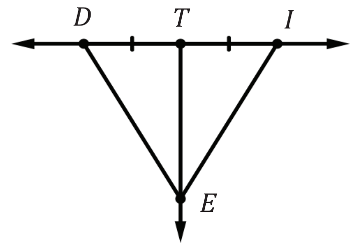
If $SI = IK = 28$, $LK = 13$, and $\overline{LI} \perp \overline{SK}$, then determine the $m\overline{SK}$. Justify your answer.

4. Consider the figure below.



Michael and Jane want to build a kite with four equal sides. They know that KT and TE are eight inches which makes point T the midpoint of $\overline{KE}$. Describe how Michael and Jane can be absolutely certain that KI, IE, ES, and KS are all the same lengths.

5. Consider the figure to the right where $\overline{DI} \perp \overline{ET}$.



If $ED = \frac{3}{4}x - \frac{5}{8}$ and $IE = \frac{1}{2}x + \frac{1}{8}$, then determine the value of x.

Section 2: Angles
Student Learning Plan

Topic Number	Topic Name	Date Completed	Study Expert(s)	Check Your Understanding Score
1	Introduction to Angles – Part 1			
2	Introduction to Angles – Part 2			
3	Angle Pairs – Part 1			
4	Angle Pairs – Part 2			
5	Special Types of Angle Pairs Formed by Transversals and Non-Parallel Lines			
6	Special Types of Angle Pairs Formed by Transversals and Parallel Lines – Part 1			
7	Special Types of Angle Pairs Formed by Transversals and Parallel Lines – Part 2			
8	Perpendicular Transversals			
9	Proving Angle Relationships in Transversals and Parallel Lines			
10	Copying Angles and Constructing Angle Bisectors			
11	Introduction to Polygons			
12	Angles of Polygons			
13	Angles of Other Polygons			

What did you learn in this section? What questions do you still have?
Who was your favorite Study Expert for this section? Why?

Section 2 – Topic 1
Introduction to Angles – Part 1

1. What is the difference between complementary and supplementary angles?

2. Suppose $m\angle TOK = 49^\circ$.

   Part A: What is the measure of the angle complement of $\angle TOK$?

   Part B: What is the measure of the angle supplement of $\angle TOK$?

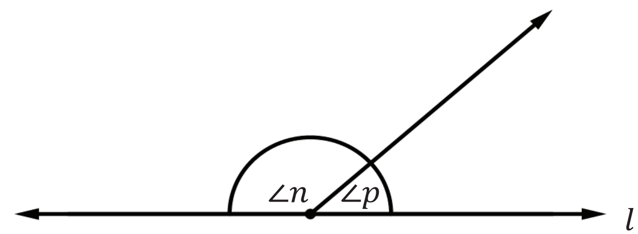
3. Suppose $m\angle YES = 113^\circ$.

   Part A: What is the measure of the angle complement of $\angle YES$?

   Part B: What is the measure of the angle supplement of $\angle YES$?

4. Angle Z is 21 degrees larger than twice the measure of angle T. If $\angle Z$ and $\angle T$ are supplementary, what is the measure of angle T?

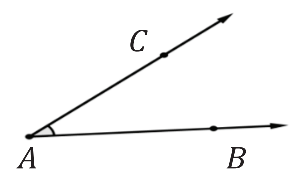
5. On line l, $m\angle p = 21x + 25$ and $m\angle n = \frac{675x-27}{3}$.



   Part A: Determine the value of x.

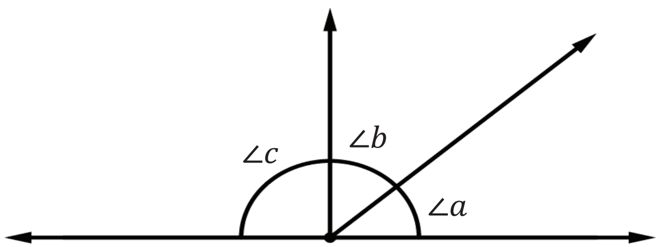
   Part B: Determine the measure of $\angle p$ and $\angle n$ in degrees.

6. Consider the following figure.



   The measure of the supplement of $\angle ABC$ is $(18x + 22)$ degrees. The measure of the complement of $\angle ABC$ is $(9x - 5)$ degrees. Determine $m\angle ABC$.

7. In the figure below, $m\angle a = 3x + 5$, $m\angle b = 5x - 18$, and $m\angle c = 7x - 2$.



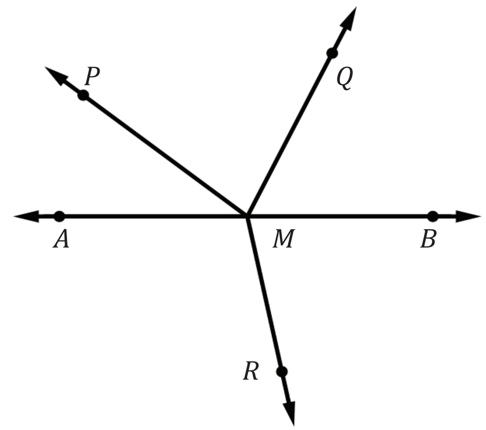
Part A: Determine $m\angle a$, $m\angle b$, and $m\angle c$.

Part B: Are $\angle a$ and $\angle b$ complementary? Justify your answer.

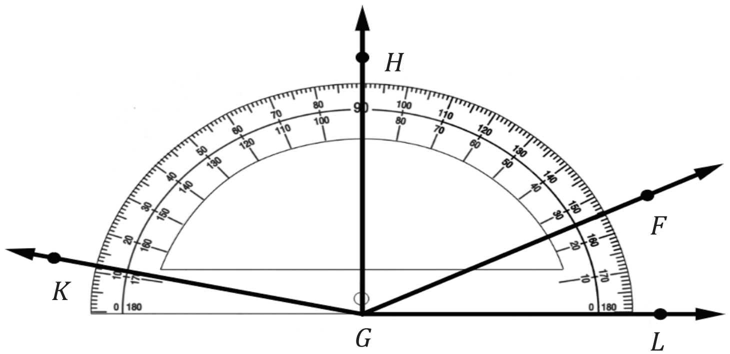
Section 2 – Topic 2
Introduction to Angles – Part 2

1. What is the measure of an angle if its two rays share the same endpoints and coincide? Provide an example.

2. Use a protractor to measure the following angles and classify each angle as obtuse, right, acute, straight, or reflex.



3. Use the figure below to fill in the blanks that define angles $\angle FGK, \angle FGH$, and $\angle KGH$ as acute, obtuse, right or straight.



$\angle FGL$ is a(n) ________________ angle.

$\angle HGF$ is a(n) ________________ angle.

$\angle KGH$ is a(n) ________________ angle.

$\angle KGF$ is a(n) ________________ angle.

$\angle KGL$ is a(n) ________________ angle.

$\angle HGL$ is a(n) ________________ angle.

4. If an angle is obtuse, what type of angle is its supplement? Justify your answer.

5. Determine whether the complement of an angle can be obtuse. Justify your answer.

6. Construct $\overline{AB}$ and label midpoint P on $\overline{AB}$. Then, construct $\overrightarrow{PQ}$. Use the space provided below.

Part A: What is the measure of $\angle QPA$?

Part B: What is the measure of $\angle QPB$?

Part C: $\angle QPA$ is
- ○ acute
- ○ obtuse
- ○ right
- ○ straight

and $\angle QPB$ is
- ○ acute.
- ○ obtuse.
- ○ right.
- ○ straight.

7. Circle the best answer that completes each statement below. Justify your answer under each statement with an example or counter example.

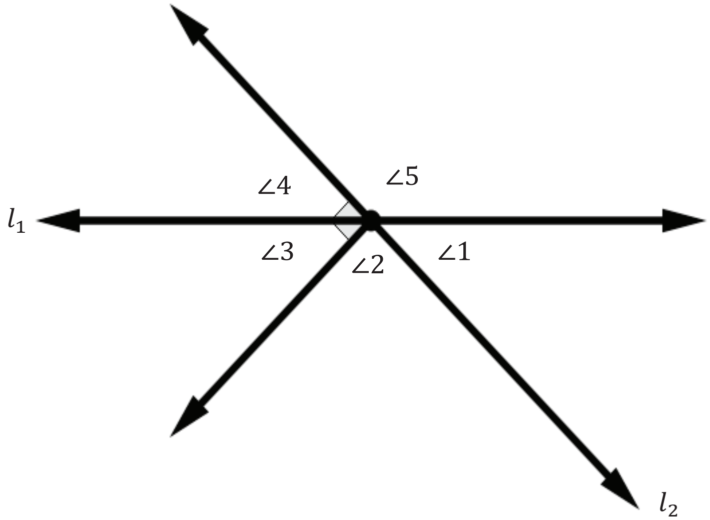
The sum of two acute angles always | sometimes | never results in an obtuse angle.

Two obtuse angles are always | sometimes | never supplementary.

The sum of two right angles always | sometimes | never results in a straight angle.

Section 2 – Topic 3
Angle Pairs – Part 1

1. Consider the figure below where l_2 intersects l_1 and $m\angle 2 = 90^\circ$.

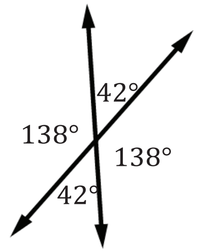


Which of the following statements are correct? Select all that apply.

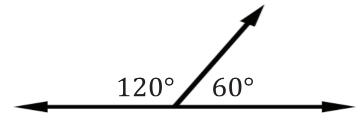
- ☐ $\angle 1$ and $\angle 4$ are adjacent angles.
- ☐ $\angle 1$ and $\angle 2$ are complementary angles.
- ☐ $\angle 3$ and $\angle 4$ are adjacent angles and complementary angles.
- ☐ $\angle 5$ is a vertical angle to the combination of $\angle 3$ and $\angle 2$.
- ☐ $\angle 1$ and $\angle 3$ are vertical angles.
- ☐ $\angle 4$ and $\angle 5$ are adjacent angles, supplementary angles, and form a linear pair.
- ☐ There is at least one angle bisector in the above graph.

2. Which of the following figures display an angle bisector?

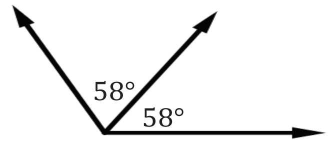
Ⓐ



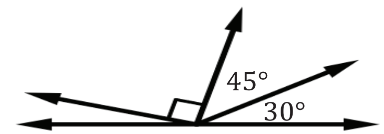
Ⓑ



Ⓒ



Ⓓ



3. Suppose that $\angle MAP$ and $\angle MAC$ are linear pairs, $m\angle MAP = 7x - 13$ and $m\angle MAC = 3x + 13$.

Part A: Identify the line and the rays that form $\angle MAP$ and $\angle MAC$.

Part B: Determine $m\angle MAP$.

Part C: Determine $m\angle MAC$.

4. Suppose that $\angle COP$ and $\angle TOD$ are vertical angles, $m\angle COP = 11x - 17$ and $m\angle TOD = 9x + 11$.

   Part A: Construct $\angle COP$ and $\angle TOD$. [Hint: $\angle COD$ is an adjacent angle to $\angle COP$ and $\angle POT$ is an adjacent angle to $\angle TOD$.]

   Part B: Determine $m\angle COD$ and $m\angle POT$.

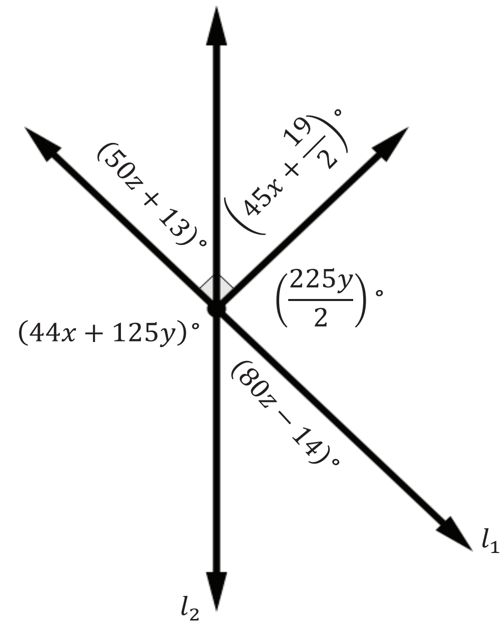
5. Suppose that $\angle LAP$ and $\angle LAR$ are adjacent angles, $m\angle LAP = 3x + 7$, $m\angle LAR = 4(x - 4)$, and $m\angle PAR = 2(3x + 7)$.

   Part A: Determine $m\angle LAP$ and $m\angle LAR$.

   Part B: What can you conclude about $\overrightarrow{AL}$? Justify your answer.

6. Is the perpendicular bisector of a line segment also an angle bisector? Justify your answer.

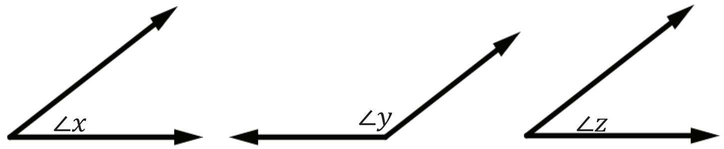
7. Consider the figure below.



The angle measures are represented by algebraic expressions. Determine the values of x, y, and z.

Section 2 – Topic 4
Angle Pairs – Part 2

1. Complete the following two-column proof that proves the Congruent Supplements Theorem.

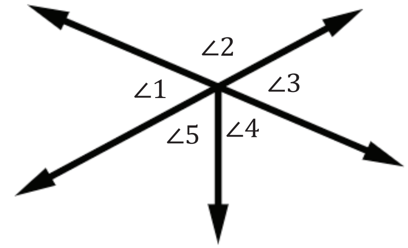


Given: $\angle x$ and $\angle y$ are supplements and $\angle y$ and $\angle z$ are supplements.

Prove: $\angle x \cong \angle z$

Statements	Reasons
1. $\angle x$ supplement to $\angle y$	**1.**
2. $m\angle x + m\angle y = 180°$	**2.**
3. $\angle y$ supplement to $\angle z$	**3.**
4. $m\angle y + m\angle z = 180°$	**4.**
5. $m\angle x + m\angle y = m\angle y + m\angle z$	**5.**
6. $m\angle x = m\angle z$	**6.**
7. $\angle x \cong \angle z$	**7.**

2. Consider the figure below.



Given: $\angle 2$ and $\angle 3$ form a linear pair, $\angle 1$ and $\angle 3$ are vertical angles, and $m\angle 1$, $m\angle 4$, and $m\angle 5$ form a straight angle.

Prove: $m\angle 2 = m\angle 4 + m\angle 5$

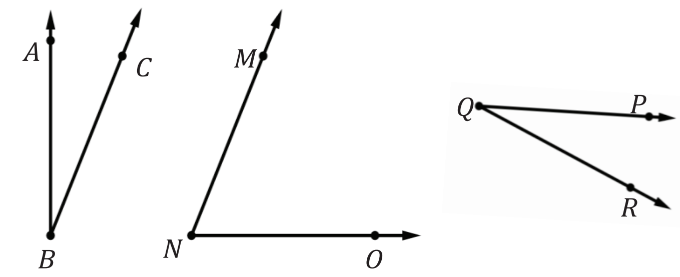
Statements	Reasons
1.	**1.**
2.	**2.**
3.	**3.**
4.	**4.**
5.	**5.**
6	**6.**
7.	**7.**
8.	**8.**

3. If $m\angle TRI + m\angle CRE = 180°$ and $\angle TRI \cong \angle CRE$, what can you conclude about $m\angle TRI$ and $m\angle CRE$? Justify your answer with a paragraph proof.

4. $\angle RIO$ and $\angle RIE$ are supplementary. One angle measures three times the other angle. What is the complement of the smaller angle?

5. The measure of an angle is five degrees greater than six times its supplement. What is the measure of the larger angle?

6. Write a plan and a two-column proof to prove the Congruent Complements Theorem using the figure below.

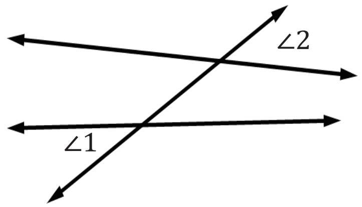


Plan:

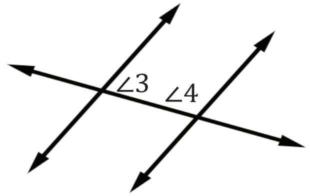
Statements	Reasons
1.	**1.**
2.	**2.**
3.	**3.**
4.	**4.**
5.	**5.**
6	**6.**

Section 2 – Topic 5
Special Types of Angle Pairs Formed by Transversals and Non-Parallel Lines

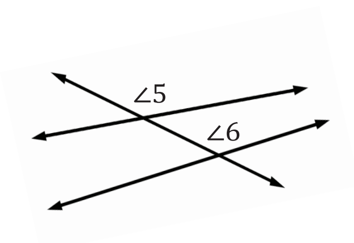
1. Classify the pair of the numbered angles. Select the correct choice under each figure.



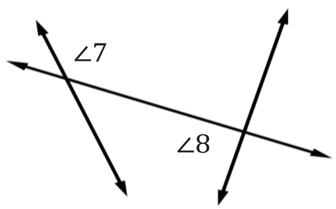
- o Alternate Exterior Angles
- o Alternate Interior Angles
- o Corresponding Angles
- o Consecutive Interior Angles



- o Alternate Exterior Angles
- o Alternate Interior Angles
- o Corresponding Angles
- o Consecutive Interior Angles

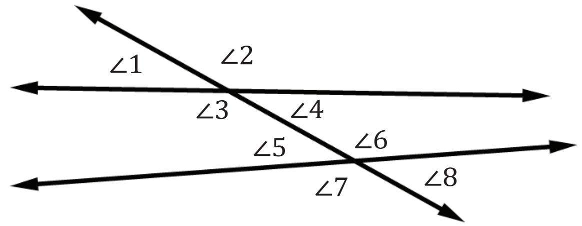


- o Alternate Exterior Angles
- o Alternate Interior Angles
- o Corresponding Angles
- o Consecutive Interior Angles



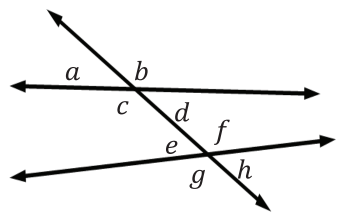
- o Alternate Exterior Angles
- o Alternate Interior Angles
- o Corresponding Angles
- o Consecutive Interior Angles

2. Identify the relationship between each pair of angles, if any.



Pair of Angles	Relationship	Pair of Angles	Relationship
∠1 and ∠7		∠4 and ∠6	
∠1 and ∠8		∠4 and ∠7	
∠2 and ∠4		∠5 and ∠4	
∠3 and ∠5		∠8 and ∠5	
∠2 and ∠6		∠8 and ∠7	

3. Identify at least one pair of the following angles.



Alternate Exterior Angles: ____________

Alternate Interior Angles: ____________

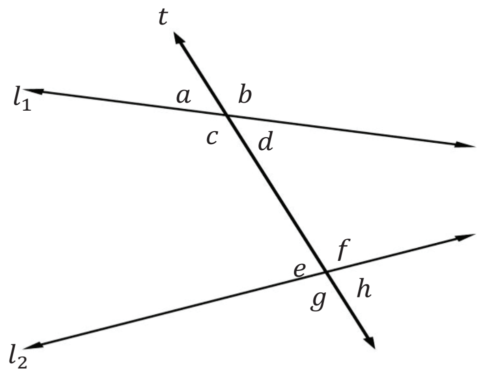
Consecutive Interior Angles: ____________

Corresponding Angles: ____________

Linear Pairs: ____________

Vertical Angles: ____________

4. Consider the figure below.



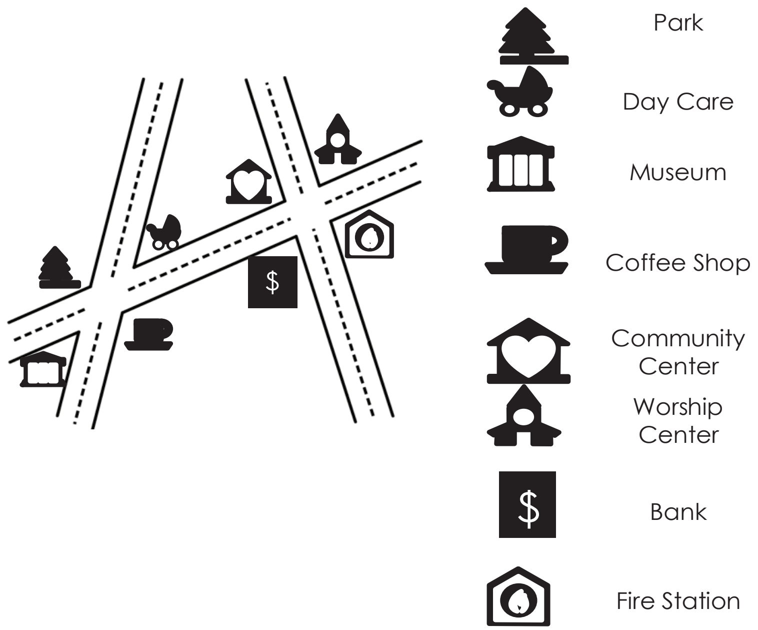
Which of the following statements is true?

- ☐ $\angle a$ and $\angle c$ lie on the same side of the transversal and form a linear pair.
- ☐ $\angle a$ and $\angle d$ are opposite sides of the transversal and form a pair of alternate interior angles.
- ☐ $\angle a$, $\angle b$, $\angle g$ and $\angle h$ are exterior angles.
- ☐ $\angle b$ and $\angle h$ lie on the same side of the transversal and form a pair of alternate exterior angles.
- ☐ $\angle d$ and $\angle f$ are consecutive interior angles lying on the same side of the transversal.
- ☐ $\angle e$ and $\angle f$ are adjacent angles and form a vertical pair of angles.
- ☐ $\angle g$ and $\angle h$ are supplementary angles and on opposite sides of the transversal.

5. Determine whether the following statements are true or false. If there is a false statement, then make it a true statement.

Statement	True or False?
Vertical angles are opposite angles with the same vertex.	O True O False
Consecutive interior angles have corresponding positions in the same side of the transversal.	O True O False
Alternate exterior angles are angles on alternate sides and between the two parallel or non-parallel lines.	O True O False
A linear pair is a set of adjacent angles that make a straight line.	O True O False

6. Consider the figure below.

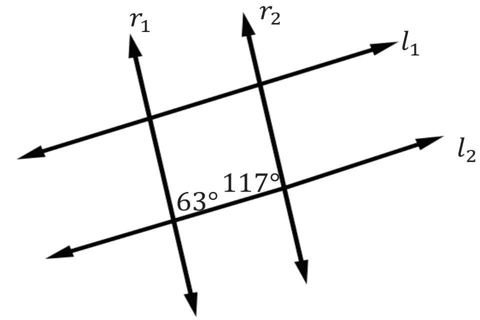


Match the angles on the left with their corresponding names on the right. Write the letter of the most appropriate answer beside each angle pair below.

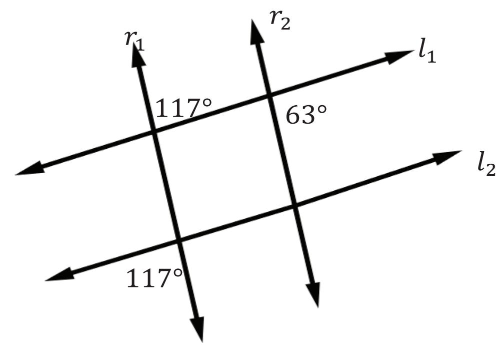
____	Museum and Worship Center	A.	Alternative Interior Angles
____	Bank and Day Care	B.	Consecutive Interior Angles
____	Park and Coffee Shop	C.	Corresponding Angles
____	Fire Station and Worship Center	D.	Vertical Angles
____	Community Center and Day Care	E.	Alternate Exterior Angles
____	Bank and Museum	F.	Linear Pair

Section 2 – Topic 6
Special Types of Angle Pairs Formed by Transversals and Parallel Lines – Part 1

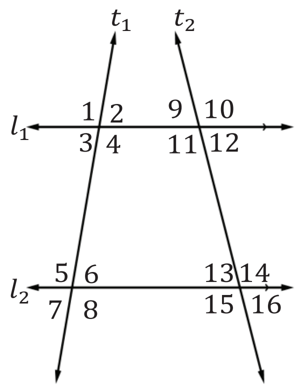
1. Consider the figure below. Which of the following lines are parallel? Justify your answer.



2. Consider the figure below. Which of the following lines are parallel? Justify your answer.



3. Consider the figure below, where l_1 and l_2 are parallel and cut by transversals t_1 and t_2.



Part A: What is the relationship between $\angle 1$ and $\angle 3$? Justify your answer.

Part B: What is the relationship between $\angle 2$ and $\angle 7$? Justify your answer.

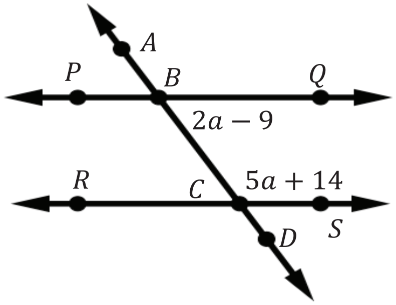
Part C: What is the relationship between $\angle 4$ and $\angle 6$? Justify your answer.

Part D: What is the relationship between $\angle 11$ and $\angle 15$? Justify your answer.

Part E: What is the relationship between $\angle 12$ and $\angle 13$? Justify your answer.

Part F: What is the relationship between $\angle 14$ and $\angle 15$? Justify your answer.

4. Consider the figure below were line $\overleftrightarrow{PQ}$ is parallel to line $\overleftrightarrow{RS}$.



Part A: Solve for a and justify your answer.

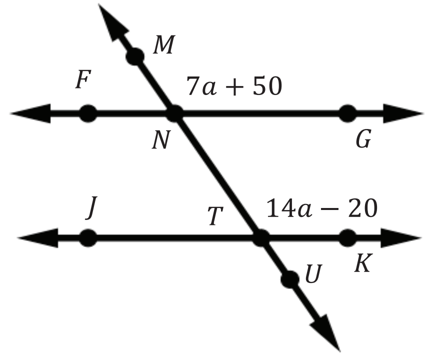
Part B: Determine $m\angle ABQ$.

Part C: Determine $m\angle BCR$.

5. Write the converse of the statement below. Then determine whether the statement is true or false. If false, give a counterexample.

   Conditional Statement: *If two angles are corresponding in parallel lines, then they are congruent.*

6. Consider the figure below. Line $\overleftrightarrow{FG}$ is parallel to line $\overleftrightarrow{JK}$.



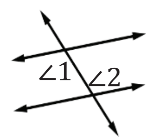
Part A: Solve for a and justify your answer.

Part B: Determine $m\angle FNT$.

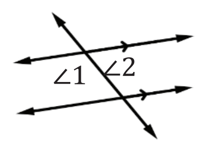
Part C: Determine $m\angle KTU$.

7. Consider the following conditional statement: *If two angles are supplementary, then they are formed by two parallel lines cut by a transversal.* Which of the following is a counterexample to this statement?

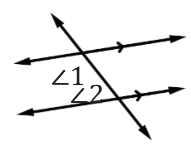
Ⓐ



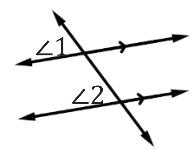
Ⓑ



Ⓒ

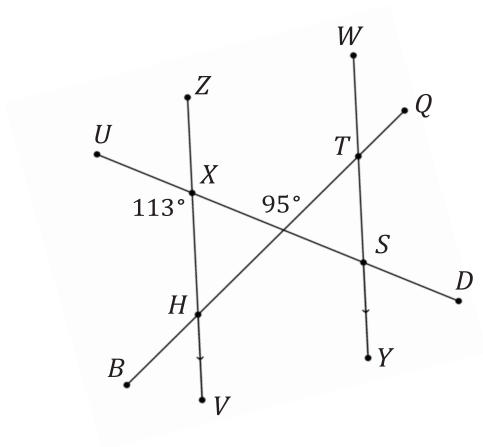


Ⓓ



Section 2 – Topic 7
Special Types of Angle Pairs Formed by Transversals and Parallel Lines – Part 2

1. Consider the figure below where $\overline{ZV}$ is parallel to $\overline{WY}$.

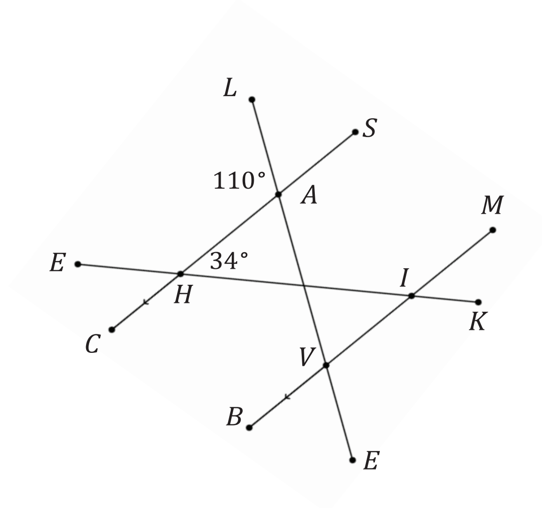


Part A: Determine $m\angle VHT$.

Part B: Determine $m\angle QTS$.

Part C: Determine $m\angle ZHQ$.

2. Consider the figure below where $\overline{CS}$ is parallel to $\overline{BM}$.

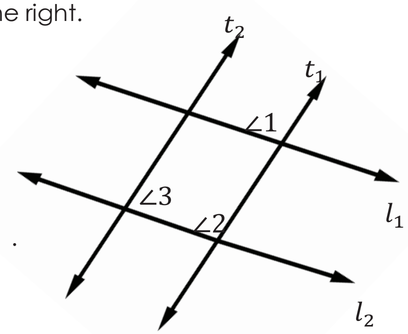


Part A: Determine $m\angle MIH$.

Part B: Determine $m\angle AVM$.

Part C: Determine the measure of the obtuse angle formed at the intersection of $\overline{AV}$ and $\overline{HI}$.

3. Consider the following diagram on the right.



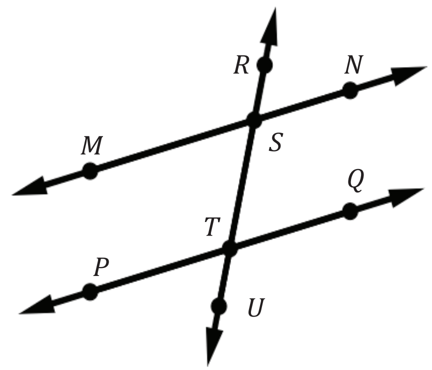
Given: $l_1 \parallel l_2$, $t_1 \parallel t_2$, and $m\angle 1 = 123^\circ$.

Prove: $m\angle 3 = 57^\circ$

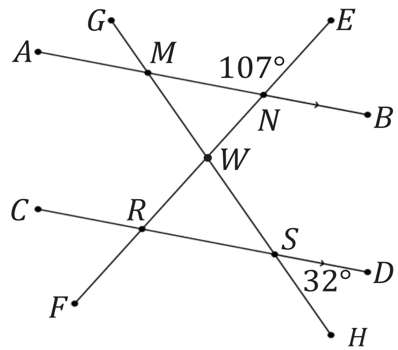
Complete the two-column proof below.

Statements	Reasons
1.	**1.**
2.	**2.**
3.	**3.**
4.	**4.**
5.	**5.**
6.	**6.**

4. Consider the figure below where $\overleftrightarrow{MN} \parallel \overleftrightarrow{PQ}$, $m\angle PTS = (19x + 5)^\circ$ and $m\angle NST = (17x + 15)^\circ$. Determine $m\angle MSR$ and $m\angle STQ$.



5. Consider the following diagram.



Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $m\angle MNE = 107^\circ$ and $m\angle DSH = 32^\circ$.

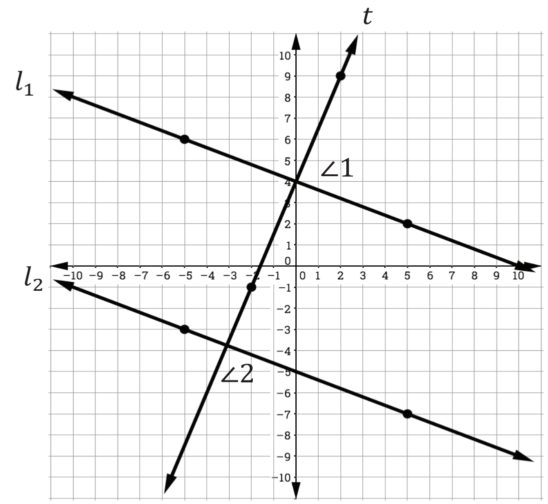
Prove: $m\angle MWR = 105^\circ$

Complete the two-column proof below.

Statements	**Reasons**
1.	**1.**
2.	**2.**
3.	**3.**
4.	**4.**
5.	**5.**
6.	**6.**
7.	**7.**
8.	**8.**

Section 2 – Topic 8
Perpendicular Transversals

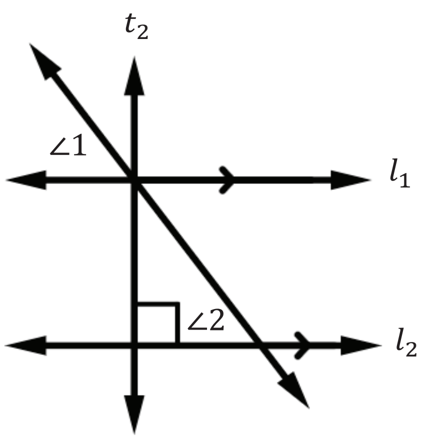
1. Consider the lines and the transversal drawn in the coordinate plane below.



Part A: Prove that $\angle 1 \cong \angle 2$. Justify your work.

Part B: Prove that $m\angle 1 = m\angle 2 = 90^\circ$. Justify your work.

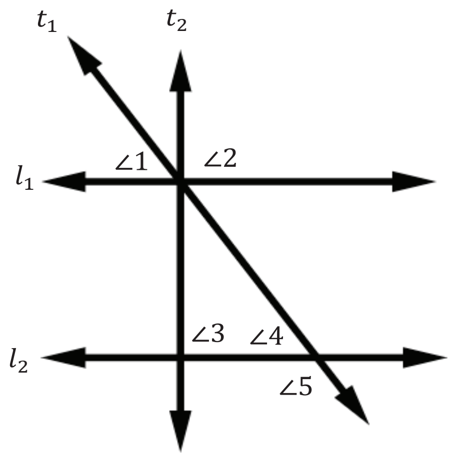
2. Consider the figure below.



Part A: If $l_1 \parallel l_2$, then use the Perpendicular Transversal Theorem to prove that $\angle 1 \cong \angle 2$. Write your answer in a paragraph proof.

Part B: Suppose your friend also proved correctly that $\angle 1 \cong \angle 2$. The difference is that your friend did not use the Perpendicular Transversal Theorem. Determine how your friend was able to prove the same statement using a different approach.

3. Consider the figure below.



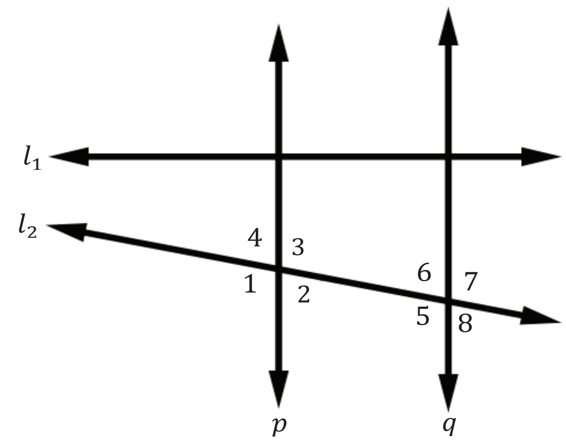
Given: $m\angle 1 = 52°$, $m\angle 2 = 90°$, and $m\angle 3 = 90°$

Prove: $m\angle 5 = 128°$

Complete the following two-column proof.

Statements	**Reasons**
1.	**1.**
2.	**2.**
3.	**3.**
4.	**4.**
5.	**5.**
6.	**6.**

4. Consider the figure below.



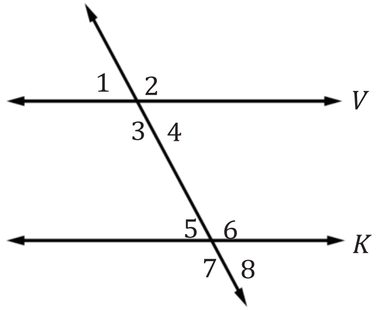
Assume that $l_1 \perp p$, $l_1 \perp q$, $m\angle 6 = (5x + 4)°$ and $m\angle 8 = (10x - 19)°$.

Part A: Determine the value of x.

Part B: Prove theoretically and algebraically that $m\angle 4 + m\angle 7 = 180$.

Section 2 – Topic 9
Proving Angle Relationships in Transversals and Parallel Lines

1. Complete the two – column proof below.



Given: $V \parallel K$

Prove: $\angle 3$ and $\angle 8$ are supplementary

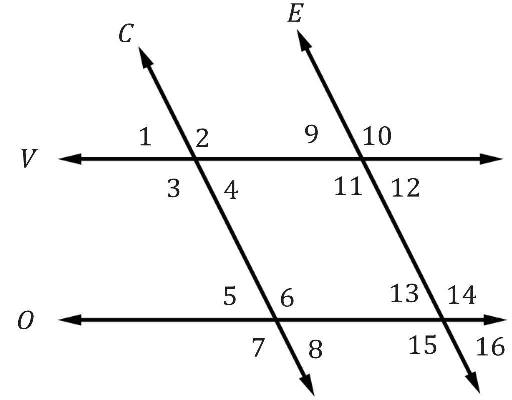
Statements	Reasons
1. $V \parallel K$	**1.**
2. $\angle 3$ and $\angle 5$ are supplementary	**2.**
3. $m\angle 3 + m\angle 5 = 180$	**3.**
4. $\angle 5 = \angle 8$	**4.**
5. $m\angle 3 + m\angle 8 = 180$	**5.**
6. $\angle 3$ and $\angle 8$ are supplementary	**6.**

2. With the image from Number 1, what type of angles are $\angle 1$ and $\angle 8$ and how are they related?

3. Complete a paragraph proof for the following.

Given: $V \parallel O$ and $C \parallel E$.

Prove: $\angle 15 \cong \angle 3$



4. Using the image from Number 3, determine the types of angles that angle 12 and 6 form and describe their relationship.

5. Revisit the proof of Number 3. Determine if the proof would be possible to prove if only one pair of lines was given to be parallel instead of both lines parallel (i.e. $V \parallel O$ and $C \nparallel E$).

Section 2 – Topic 10
Copying Angles and Constructing Angle Bisectors

1. Reorder the steps to construct $\angle JOB$, a copy of $\angle A$.

_______ Construct $\overrightarrow{OJ}$.

_______ Place your compass point at the vertex of $\angle A$. Create an arc that intersects both rays of $\angle A$.

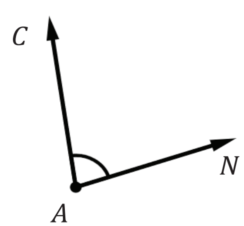
_______ On $\angle A$, set your compass point on the intersection of the arc and ray and the pencil on the other intersection of the arc and second ray. Lock your compass.

_______ Place the point of the compass on the intersection of the arc and $\overrightarrow{OB}$. Mark an arc through the large arc created in a previous step. Label the point of intersection of the two arcs point J.

_______ Draw a ray that will become one of the two rays of the new angle. Label the ray $\overrightarrow{OB}$.

_______ Without changing your compass setting, create an arc from point O that intersects $\overrightarrow{OB}$. Be sure to make a large arc.

2. Construct $\angle DLE$ to be a copy of $\angle CAN$ below.



3. Reorder the steps to construct $\overrightarrow{AL}$, an angle bisector of $\angle FAM$.

_______ Place the point of the compass on the intersection of the arc and the ray, then draw an arc in the interior of the angle.

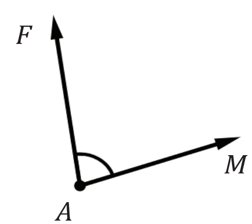
_______ Place the point of the compass on the angle's vertex.

_______ Using a straightedge, construct a ray from the vertex A through the point where the arcs intersect, L.

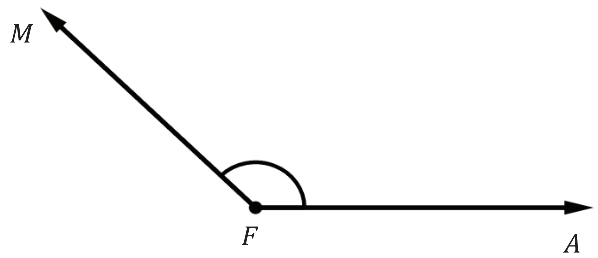
_______ Without changing the compass setting, repeat a previous step for the other angle so that the two arcs intersect in the interior of the angle. Label the intersection L.

_______ Without changing the width of the compass, draw an arc across each ray.

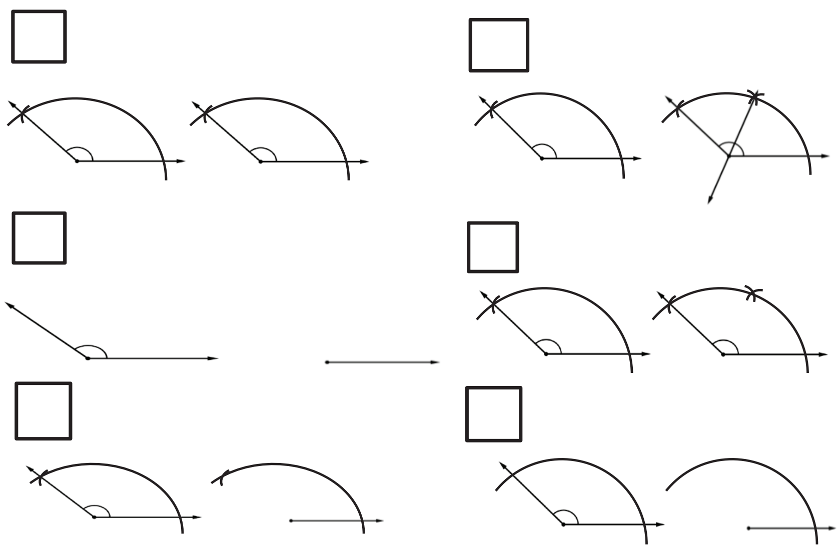
4. Construct $\overrightarrow{AL}$ to be an angle bisector of $\angle FAM$ below.



5. Consider the following angle $\angle MFA$ below.



Label each box with the correct order of copying $\angle MFA$ and constructing an angle bisector, $\overrightarrow{FS}$.



Section 2 – Topic 11
Introduction to Polygons

1. Draw an example for each of the following polygons.

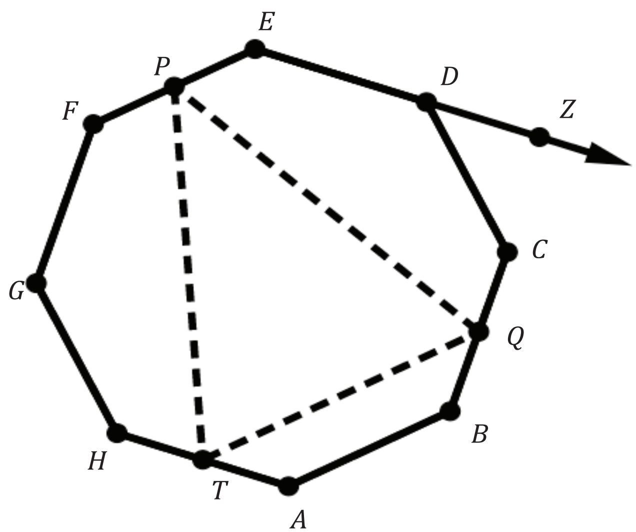
Convex polygon: irregular triangle	
Concave polygon: octagon	
Convex polygon: regular quadrilateral	

2. Suppose you have two convex polygons, one regular and the other irregular. Determine the similarities and differences between the interior and exterior angles of both polygons.

3. Suppose you have a concave pentagon with interior angles that measure $110°$, $80°$, $72°$, and $62°$ for four of its vertices. Which of the following statements are true for the concave pentagon? Select all that apply.

- ☐ The measure of the interior angle at the fifth vertex is greater than $180°$.
- ☐ One diagonal lies outside the pentagon.
- ☐ All the vertices of the pentagon point outwards.
- ☐ The sum of the exterior angles of the pentagon is greater than $360°$.
- ☐ The pentagon is regular.

4. Consider the regular octagon $ABCDEFGH$ with $\overrightarrow{EZ}$ below. Lucas cut the regular octagon into the triangle PQT.

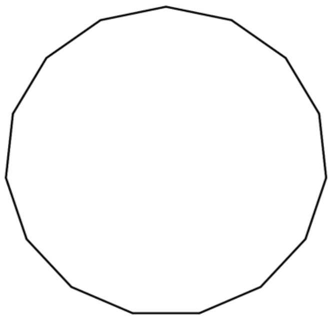


If $m\angle QPT = m\angle ZDC$ and $m\angle PQT = m\angle PTQ$, verify the sum of the interior angles of triangle PQT using two different methods to determine $m\angle ZDC$.

5. Match the terms below to their definition.

________	Circumcenter	**A.** All congruent sides and angles
________	Regular Polygon	**B.** The distance from center to the median of the side of a regular polygon
________	Incircle	**C.** The distance from the center to any vertex.
________	Radius of Polygon	**D.** The point that is equidistant from each vertex.
________	Apothem	**E.** Connects all the vertices of a regular polygon.
________	Circumcircle	**F.** No congruent sides and angles
________	Irregular Polygon	**G.** Connects all the midpoints of the inside of a regular polygon.

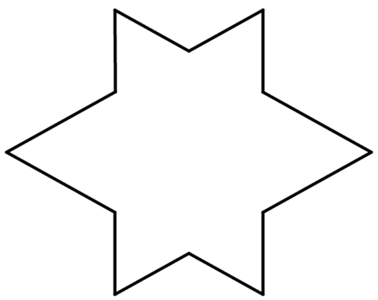
6. Consider the regular polygon below.



Part A: Draw the circumcircle, center, incircle, radii, and apothem.

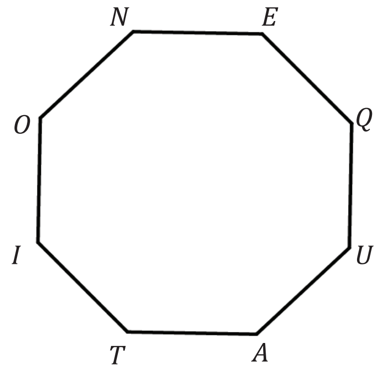
Part B: Determine how many isosceles triangles the polygon above has.

7. Consider the irregular polygon below.



Break up the polygon above into three- and four-sided shapes.

8. Consider the regular polygon $EQUATION$ below.



Which of the following statements are correct? Select all that apply.

- ☐ The diagonals $\overline{EO}$ and $\overline{EU}$ creates congruent triangles.
- ☐ The polygon $EQUATION$ has seven different isosceles triangles.
- ☐ The area of the polygon is the average of the area of the circumcircle and area of the incircle.
- ☐ The midpoints of $\overline{OI}, \overline{IT}, \overline{TA}$, and $\overline{EQ}$ create a kite.
- ☐ The radius of polygon $EQUATION$ is smaller than the apothem.

Section 2 – Topic 12
Angles of Polygons

1. What are the measures of each interior angle and each exterior angle of regular nonagon $ANJOLIQUE$?

2. The sum of the interior angles of a regular polygon is $2{,}340°$.

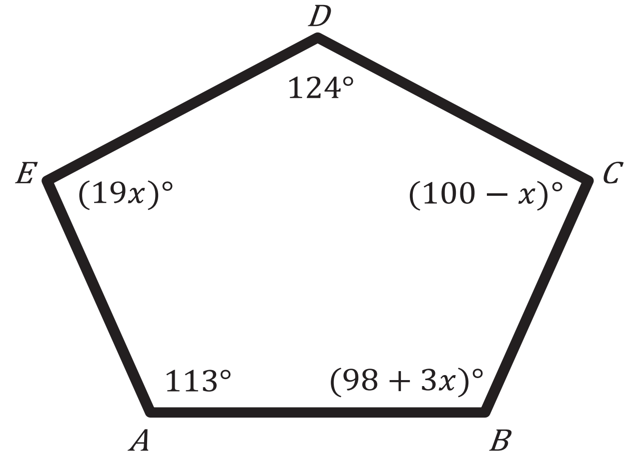
   Part A: Classify the polygon by the number of sides.

   Part B: What is the measure of one interior angle of the polygon?

   Part C: What is the measure of one exterior angle of the polygon?

3. If the measure of an exterior angle of a regular polygon is $20°$, how many sides does the polygon have? Justify your answer.

4. Consider pentagon $ABCDE$.



   Part A: Find the value of x.

   Part B: Determine $m\angle B$, $m\angle C$, and $m\angle E$.

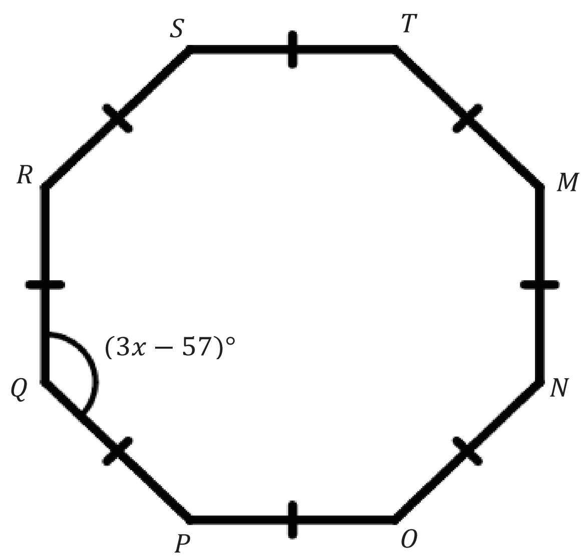
   Part C: Find the value of each exterior angle.

5. Given a regular heptagon and a regular hexagon,

   Part A: Which one has a greater exterior angle? By how much is the angle greater?

   Part B: Which one has a greater interior angle? By how much is the angle greater?

6. Consider the regular octagon below.



   Part A: Find the value of each interior angle.

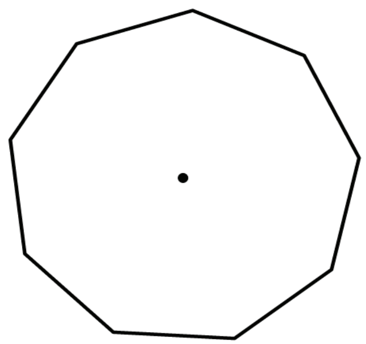
   Part B: Find the value of x.

   Part C: Find the value of each exterior angle.

7. The sum of the interior angles of a hexagon is equal to the sum of six consecutive integers. What is the measure of the smallest interior angle of the hexagon?

Section 2 – Topic 13
Angles of Other Polygons

1. Mark a central angle and calculate the measure of the central and base angles in the polygon below.

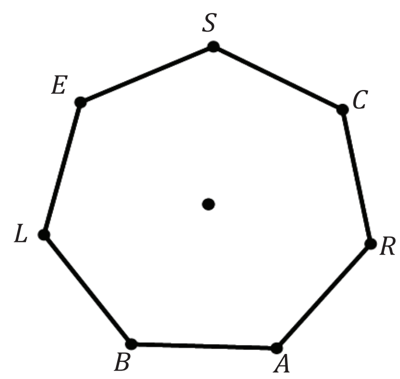


2. Determine the central and base angle of a 35-gon (triacontakaipentagon).

3. René Descartes, a founder of geometry, used a chiliagon in his discussions of philosophy. Determine the central and base angles of a chiliagon, 1000-gon.

4. Explain why the sum of the interior angles of a polygon is $180(n-2)$, where n is the number of sides.

5. Consider the regular heptagon.



Part A: Determine the measure of $\angle BLE$.

Part B: Draw a ray starting at B extending through L and create BD.

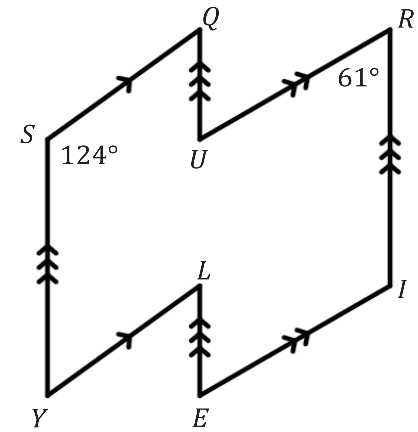
Part C: Determine the measure of $\angle ELD$.

6. If a regular polygon has a sum of $3{,}960°$ for all the interior angles, then determine the measure of one exterior angle.

7. If a regular polygon has an exterior angle of $7.2°$, then determine the sum of all the interior angles.

8. Determine the central angle and sum of exterior angles of an irregular nonagon.

9. Consider the following irregular octagon and answer the questions below it.



Part A: Determine the measure of every interior angle. Justify your answer.

Part B: Determine the measure of every exterior angle.

Section 3: Rigid Transformations and Symmetry

Student Learning Plan

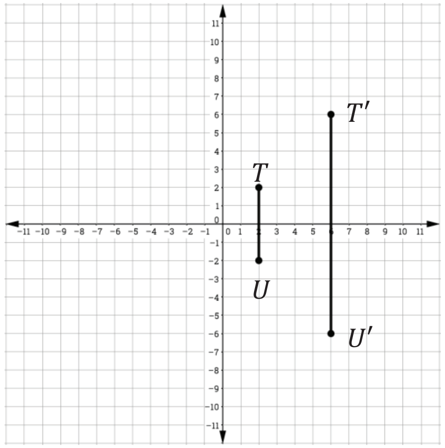
Topic Number	Topic Name	Date Completed	Study Expert(s)	Check Your Understanding Score
1	Introduction to Transformations			
2	Examining and Using Translations			
3	Translations of Polygons			
4	Examining and Using Reflections			
5	Reflections of Polygons			
6	Examining and Using Rotations			
7	Rotations of Polygons - Part 1			
8	Rotations of Polygons - Part 2			
9	Angle-Preserving Transformations			
10	Symmetries of Regular Polygons			

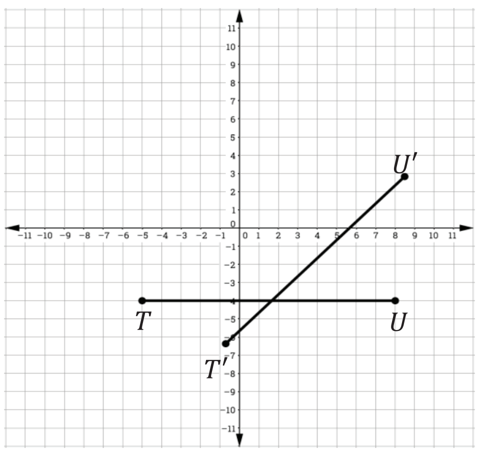
What did you learn in this section? What questions do you still have?

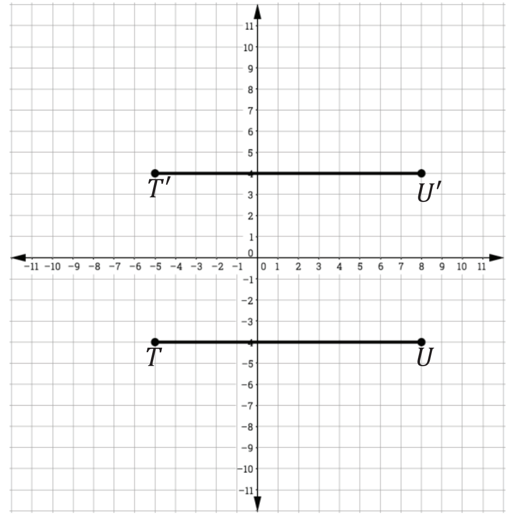
Who was your favorite Study Expert for this section? Why?

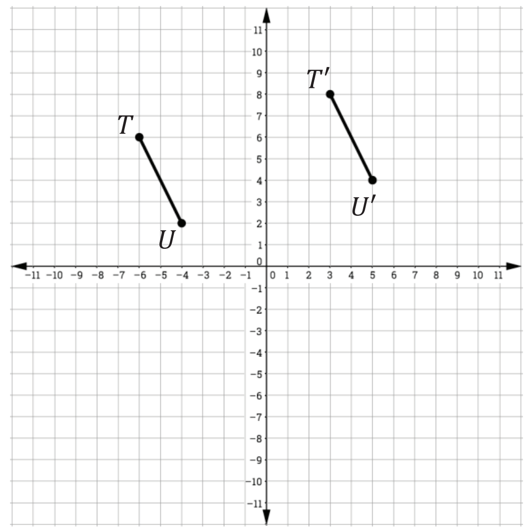
Section 3 – Topic 1
Introduction to Transformations

1. Identify the transformations shown in the following graphs and write the names of the transformations in the corresponding boxes under each graph.

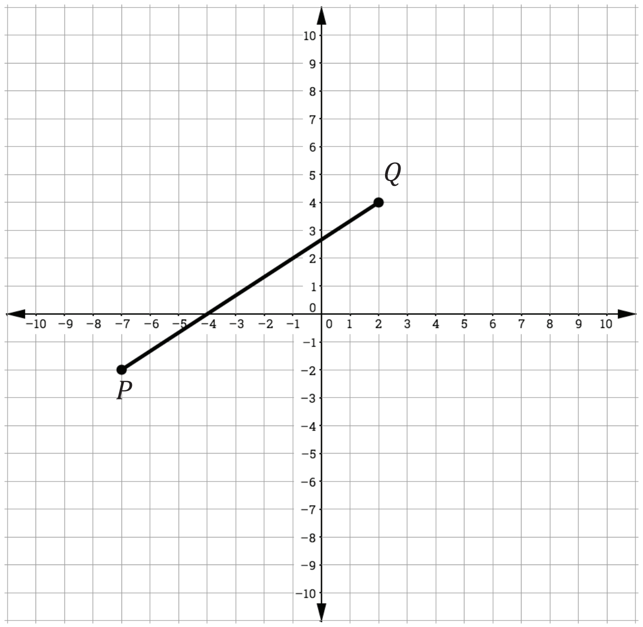








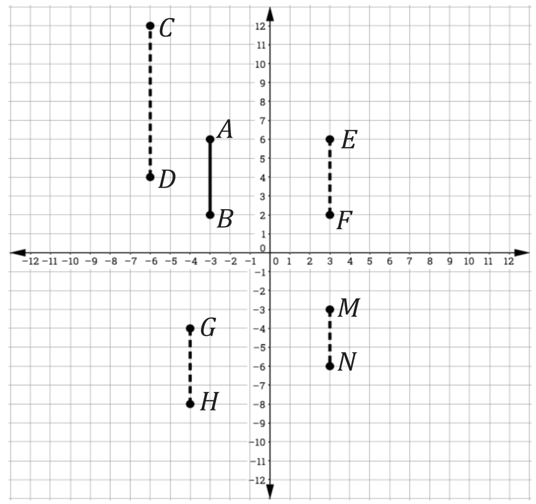
2. Consider $\overline{PQ}$ in the coordinate plane below.



Write the coordinates of P' and Q' after the following transformations.

Transformations	P'	Q'
$\overline{PQ}$ is translated three units up and five units to the right.		
$\overline{PQ}$ is rotated $180°$ clockwise about the origin.		

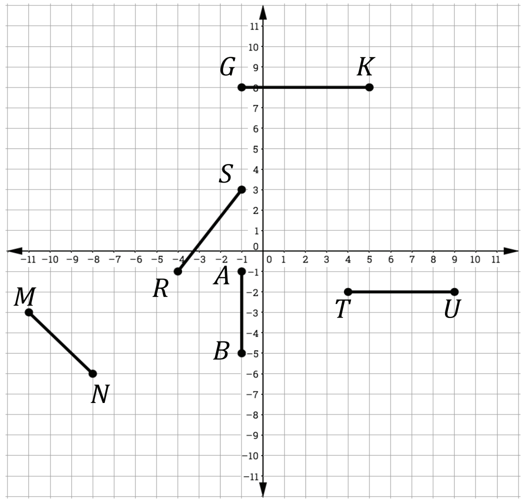
3. Consider $\overline{AB}$ in the coordinate plane below. The dashed line segments represent transformations of $\overline{AB}$.



Describe each of the following line segments as a result of a transformation of $\overline{AB}$.

	Describe the Transformation
$\overline{CD}$	
$\overline{EF}$	
$\overline{GH}$	
$\overline{MN}$	

4. Consider the graph below.



Which of the following statements are true about the graph? Select all that apply.

- ☐ $\overline{TU}$ is the result of a translation of $\overline{RS}$ followed by a rotation.
- ☐ $\overline{AB}$ is the result of a reflection of $\overline{GK}$ followed by a rotation.
- ☐ $\overline{TU}$ is the result of a dilation of $\overline{GK}$ by a scale factor less than one.
- ☐ $\overline{RS}$ is the result of a reflection of $\overline{MN}$.
- ☐ $\overline{GK}$ is the result of 90° rotation of $\overline{AB}$ followed by a translation of nine units up.
- ☐ $\overline{MN}$ is the result of a dilation of $\overline{AB}$ by a scale factor less than one.

Section 3 – Topic 2
Examining and Using Translations

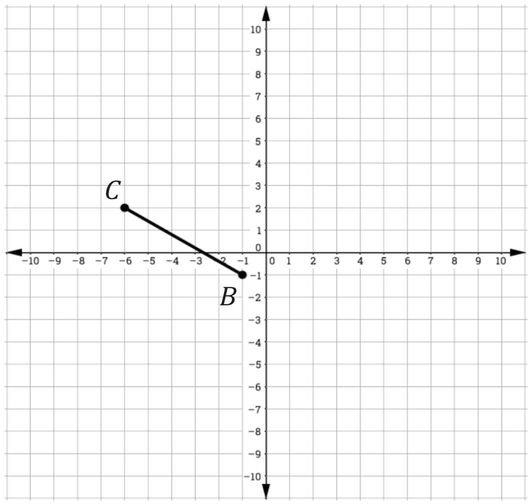
1. Do the diagrams below illustrate translations?

<table>
<tr><th>Diagrams</th><th colspan="2">Is this a translation?</th></tr>
<tr><td></td><td>O Yes</td><td>O No</td></tr>
<tr><td></td><td>O Yes</td><td>O No</td></tr>
<tr><td></td><td>O Yes</td><td>O No</td></tr>
<tr><td></td><td>O Yes</td><td>O No</td></tr>
<tr><td></td><td>O Yes</td><td>O No</td></tr>
<tr><td></td><td>O Yes</td><td>O No</td></tr>
<tr><td></td><td>O Yes</td><td>O No</td></tr>
</table>

2. What does the transformation $(x, y) \to (x + 2, y - 5)$ represent?

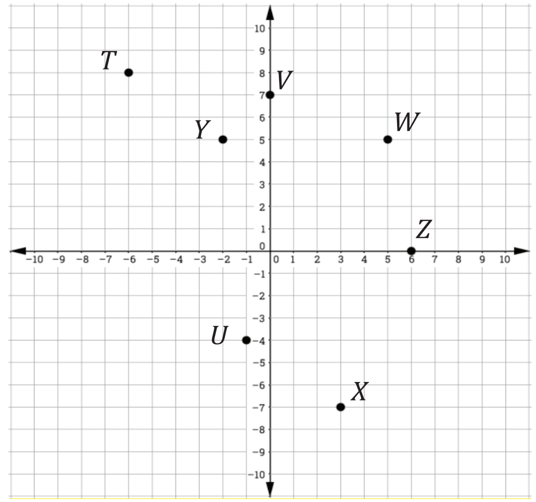
3. What is the algebraic description for a transformation that translates the point (x, y) nine units to the right and five units down?

4. Transform $\overline{BC}$ according to $(x, y) \to (x - 4, y - 1)$. Sketch and write the coordinates for $\overline{B'C'}$.



5. When the transformation $(x, y) \to (x, y + 3)$ is performed on $\overline{MP}$, its image, $\overline{M'P'}$, is on the x-axis with M' on the origin and P' on -3. What are the coordinates of $\overline{MP}$? Justify your answer.

6. Consider the following points on the coordinate plane.

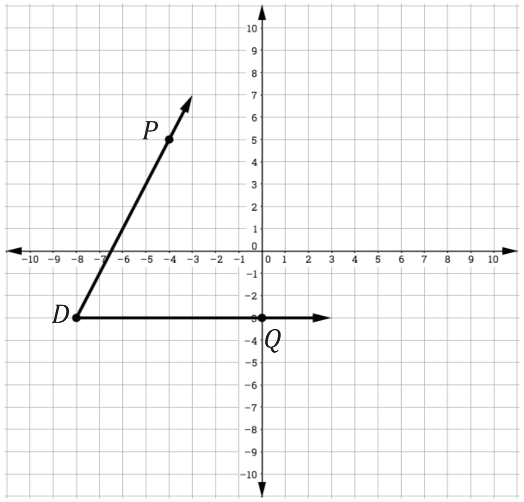


Under the translation $(x, y) \to (x + 2, y - 3)$, the points on the graph will become:

T'(_____,_____), U'(_____,_____), V'(_____,_____), W'(_____,_____), X'(_____,_____),

Y'(_____,_____), and Z'(_____,_____).

7. $\angle PDQ$ undergoes the translation $(x, y) \rightarrow (x + h, y + k)$ such that $P'(-6, -2)$, $D'(-10, -10)$, and $Q'(-2, -10)$.



Part A: What are the values of h and k?

$h =$ ________ units

$k =$ ________ units

Part B: Which of the following statements is true?

Ⓐ $\angle PDQ$ and $\angle P'D'Q'$ have different locations.
Ⓑ $\angle PDQ$ and $\angle P'D'Q'$ have different shapes.
Ⓒ $\angle PDQ$ and $\angle P'D'Q'$ have different sizes.
Ⓓ $\angle PDQ$ and $\angle P'D'Q'$ have different directions.

8. Points A, B, C, D, and E were translated. The table of translations is shown below.

Pre-Image	Image
$A(5, 7)$	
$B(-3, 9)$	$B'(-7, 3)$
	$C'(5, 2)$
$D(10, -8)$	
$E(0, 4)$	$E'(-4, -2)$

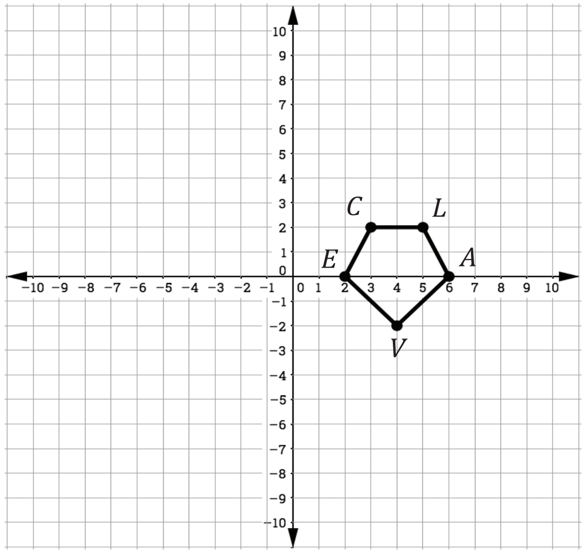
Part A: Complete the missing cells with the corresponding coordinates.

Part B: Complete the following algebraic description for the table of translations.

$(x, y) \rightarrow (x + \boxed{}, y + \boxed{})$

Section 3 – Topic 3
Translations of Polygons

1. Translate pentagon $CLAVE$ following this algebraic description $(x, y) \rightarrow (x - 4, y + 3)$. Sketch $C'L'A'V'E'$ in the coordinate plane below.

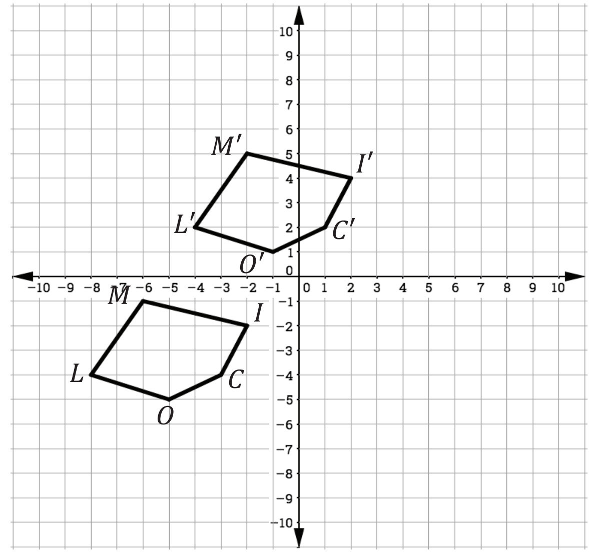


2. Quadrilateral $PUMA$ has coordinates at $P(-5, -2)$, $U(-1, 2)$, $M(4, -3)$, $A(0, -7)$ and it is transformed by $(x, y) \rightarrow (x + 5, y - 7)$.

   Part A: What is the x –coordinate of U'?

   Part B: What is the y –coordinate of M'?

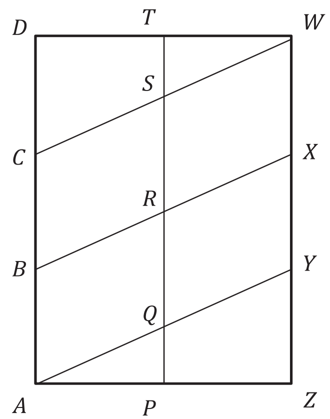
3. Consider the figure below.



Complete the algebraic description of the transformation of $MICOL$.

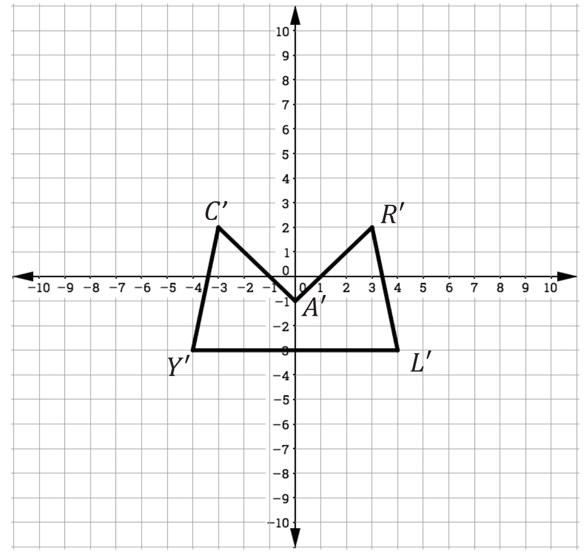
$(x, y) \rightarrow (x + \boxed{}, y + \boxed{})$

4. Consider the figure below.



If W is the image of R after a translation, then which point is the image of B under the same translation?

5. Polygon $C'A'R'L'Y'$ is the image of polygon $CARLY$ after a translation $(x, y) \rightarrow (x - 4, y - 6)$.



What are the coordinates of the vertices of polygon $CARLY$?

Section 3 – Topic 4
Examining and Using Reflections

1. Determine whether the following statements are true or false. Justify your answer.

Statements	True or False?	
The image of the point $(7,-2)$ under a reflection across the x-axis is $(-7,-2)$.	O True	O False
Justify your answer.		
The image of the point $(-8,-5)$ under a reflection across the y-axis is $(8,-5)$.	O True	O False
Justify your answer.		
The image of the point $(4,3)$ under a reflection across the line $y=-x$ is $(-3,-4)$.	O True	O False
Justify your answer.		
The image of the point $(-1,-5)$ under a reflection across the $y=x$ is $(-7,-3)$.	O True	O False
Justify your answer.		
Point $A(1,3)$ is reflected across the x-axis to create point A'. The line $y=2x-5$ pass through point A'.	O True	O False
Justify your answer.		

2. What is the difference in the rule of reflection for a point reflected across $y=x$ and a point reflected across $y=-x$?

3. Suppose the line segment whose endpoints are $H(5,0)$ and $I(-6,-3)$ is reflected over the y-axis. What are the coordinates $H'I'$?

4. Suppose a line segment whose endpoints are $G(-1,-7)$ and $L(2,6)$ is reflected over $y=-x$. What are the coordinates of $\overline{G'L'}$?

5. Suppose the line segment whose endpoints are $P(0,-4)$ and $D(6,1)$ is reflected over the x-axis, and then reflected again over $y=x$. What are the coordinates P" and D"?

6. Suppose a line segment whose endpoints are $T(x_1, y_1)$ and $U(x_2, y_2)$ is reflected over $y = x$ to create $\overline{T'U'}$. The coordinates of the image are $T'(-4, 3)$ an $U'(1, 7)$. What are the values of x_1, x_2, y_1, and y_2?

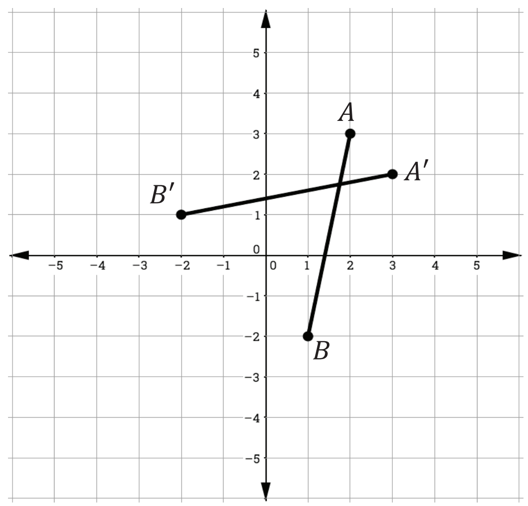
7. Consider the following points.

   $Y(-4, -1)$ and $O(4, 2)$

   Let $\overline{Y'O'}$ be the image of $\overline{YO}$ after a reflection across line r. Suppose that Y' is located at $(1, 4)$ and O' is located at $(-2, -4)$. Which of the following is true about line r?

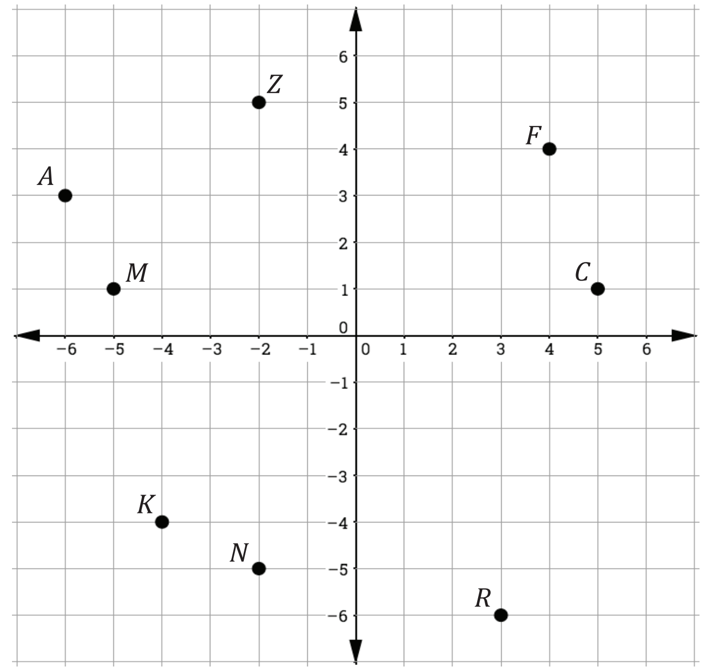
   Ⓐ Line r is represented by the x-axis.
   Ⓑ Line r is represented by the y-axis.
   Ⓒ Line r is represented by $y = x$.
   Ⓓ Line r is represented by $y = -x$.

8. Consider the following graph.



   Determine the type of reflection performed above.

9. Consider the following graph.



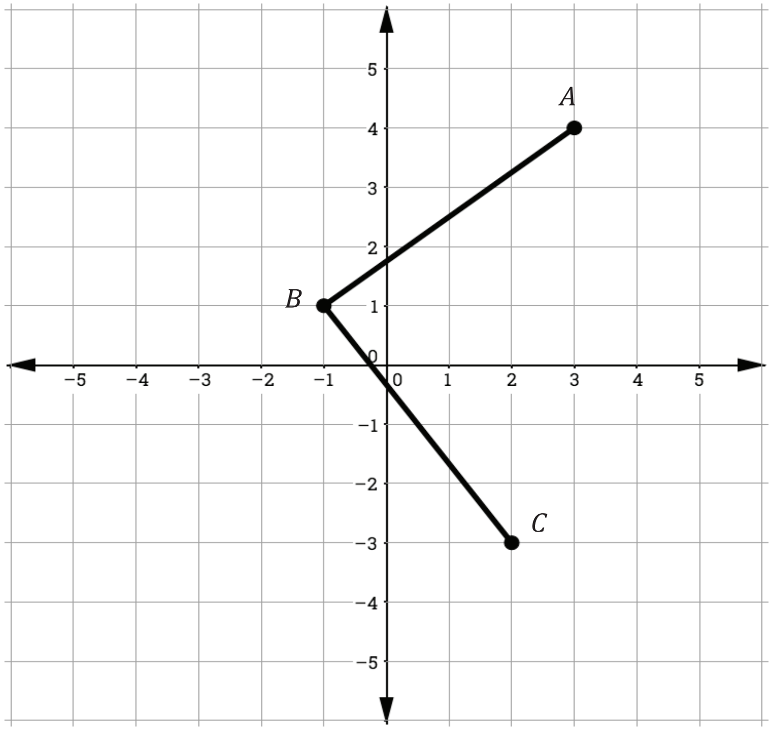
   Part A: Which point is a reflection of point R over the line $y = x$?

   Part B: Which point is a reflection of point C over the y-axis?

   Part C: Which point is a reflection of point Z over the x-axis?

   Part D: Which point is a reflection of point K over the line $y = -x$?

10. Consider the following graph.



Find the coordinates of figure $A'B'C'$ and the images of the vertices of figure ABC under the following reflections.

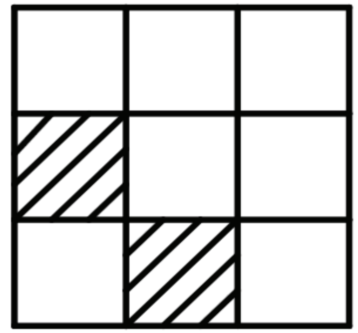
r_{x-axis}

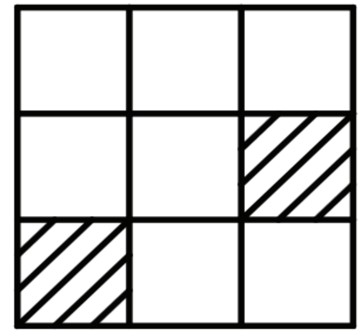
r_{y-axis}

$r_{y=x}$

Section 3 – Topic 5
Reflections of Polygons

1. Draw the line(s) of reflection of the following figures.

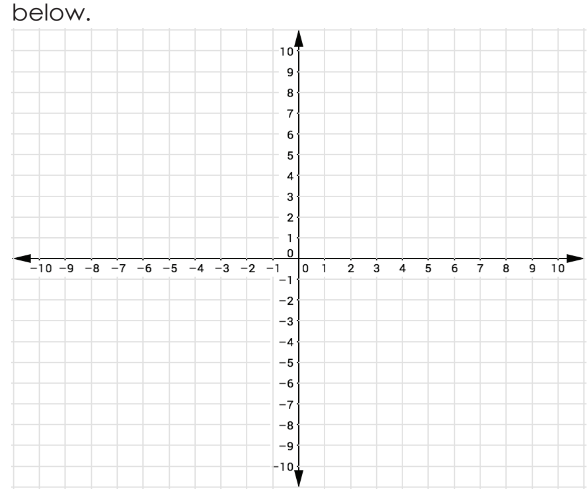




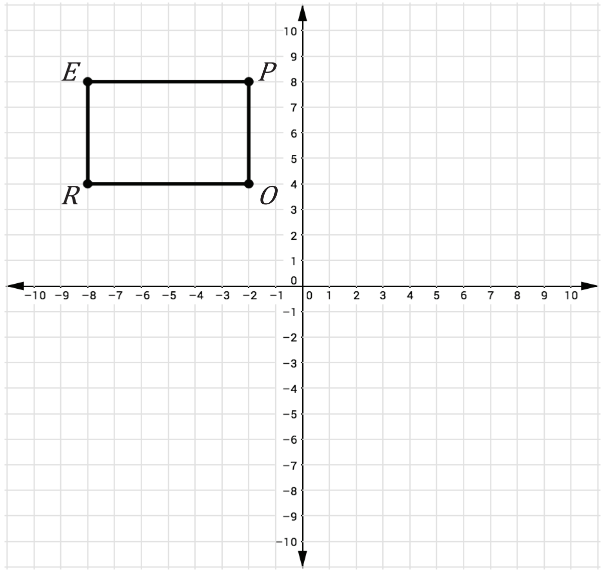
2. Quadrilateral $FRIO$ is the result of a reflection of quadrilateral $LAMB$ over the y –axis. $FRIO$ has vertices at $F(-7,6), R(1,7), I(2,-5)$, and $O(-6,-6)$.

Part A: What are the coordinates of the vertices of $LAMB$?

Part B: Graph quadrilaterals $FRIO$ and $LAMB$ on the coordinate plane below.



3. Consider rectangle $ROPE$ on the coordinate plane.

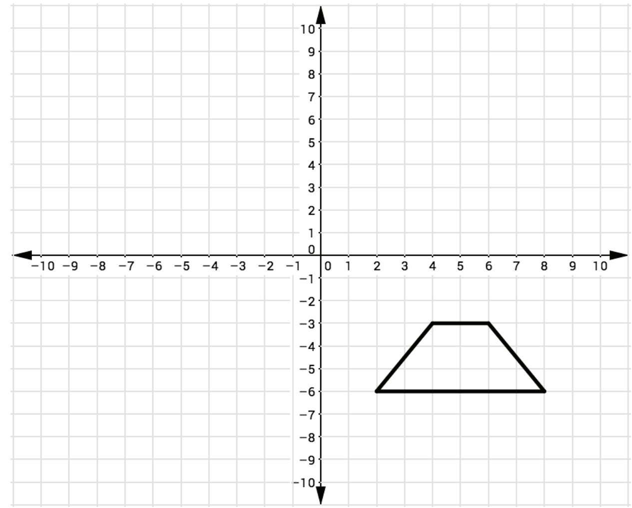


Part A: Draw a reflection over the line $y = x$. Write down the coordinates of the reflected figure.

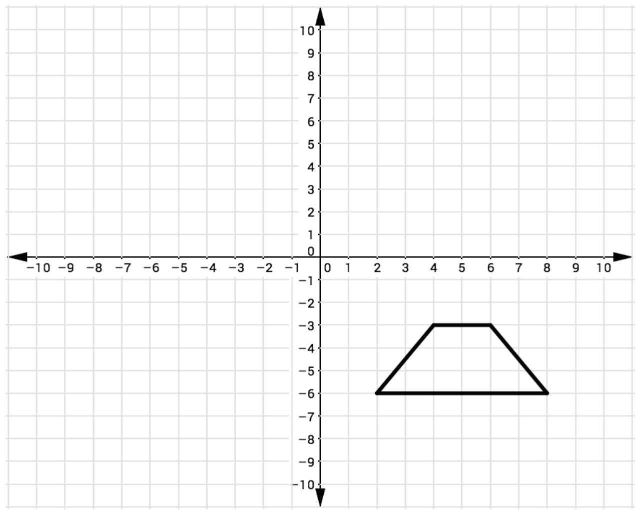
Part B: Draw a reflection over the line $y = -x$. Write down the coordinates of the reflected figure.

4. Explain why, when you reflect a point across the line $y = x$, the x –coordinate and the y –coordinate change places, and when you reflect a point across the line $y = -x$, the x –coordinate and the y –coordinate change places and their signs are changed.

5. Sketch the reflection of the following image over the y –axis and explain why the rule $(x, y) \rightarrow (-x, y)$ applies to this transformation.

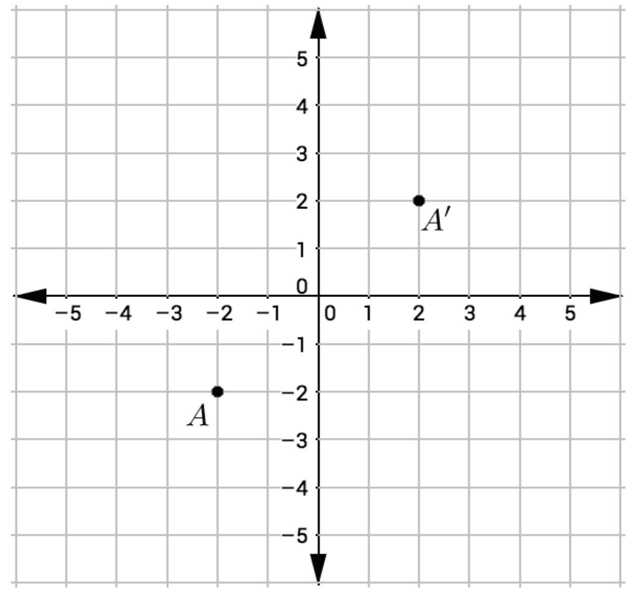


6. Edgar argues that reflecting the following image over the x –axis and then over the line $y = x$ is the same as reflecting the same image over the line $y = -x$. Prove whether Edgar is correct or not.



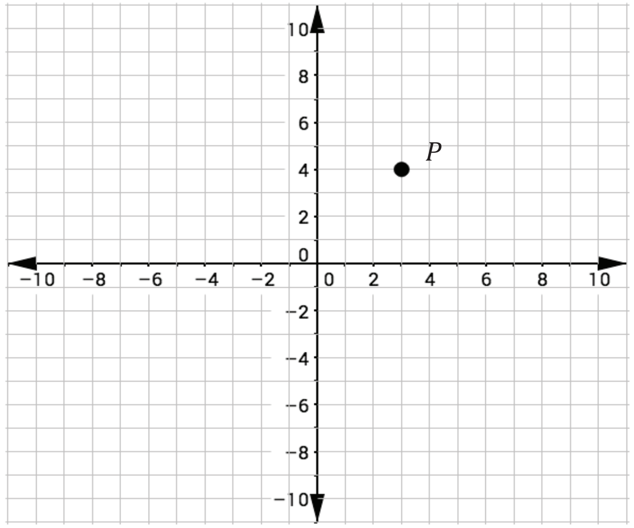
Section 3 – Topic 6
Examining and Using Rotations

1. Consider the following graph.

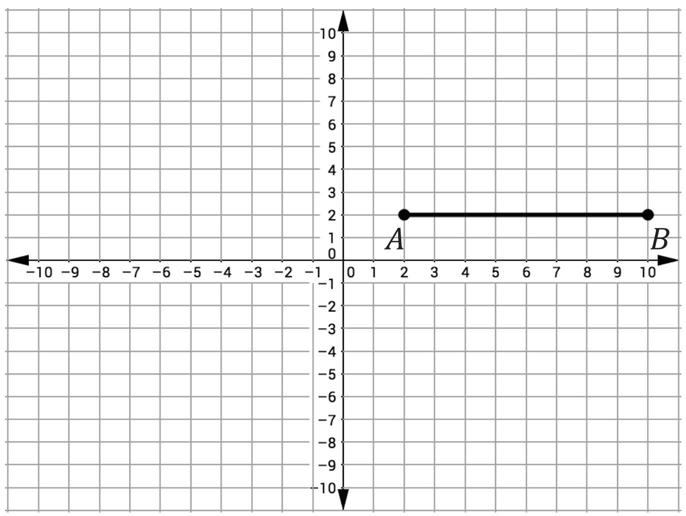


Point A' is the result of a rotation of point A. Describe the rotation.

2. Consider the following graph and rotate point P $90°$ counterclockwise around two points: the origin and $(-2, 4)$. Sketch both points on the graph.



3. Consider the following graph.



Complete the table on the next page after rotating $\overline{AB}$ $90°$, $180°$, and $270°$ counterclockwise around the origin.

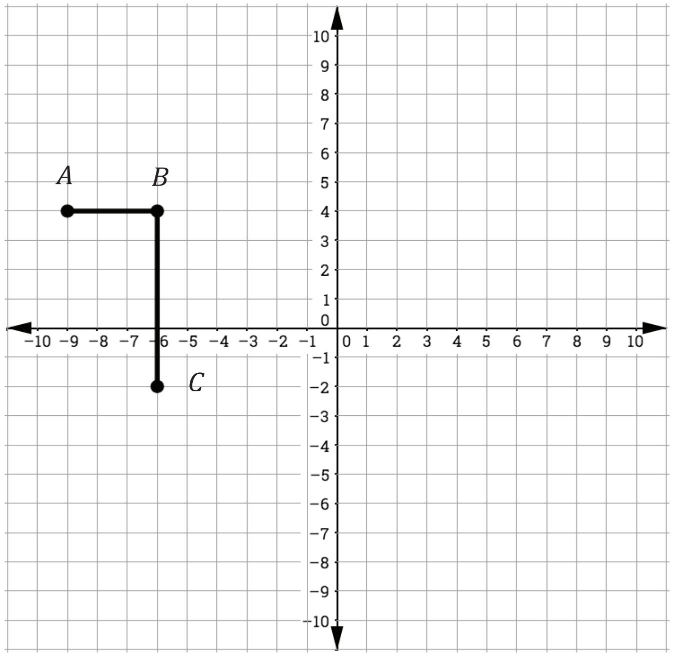
Rotation	**Counterclockwise**	
$90°$ Rotation:	$A'(x, y) \rightarrow$ ______	$B'(x, y) \rightarrow$ ______
$180°$ Rotation:	$A'(x, y) \rightarrow$ ______	$B'(x, y) \rightarrow$ ______
$270°$ Rotation:	$A'(x, y) \rightarrow$ ______	$B'(x, y) \rightarrow$ ______

4. $\overline{BK}$ has endpoints $B(1, 4)$ and $K(4, -3)$. Rotate $\overline{BK}$ clockwise 270 degrees about the origin.

   Part A: Write an algebraic description of the transformation of $\overline{BK}$.

   Part B: What are the endpoints of the new line segment?

5. Consider the following figure.



Part A: Rotate figure ABC $270°$ counterclockwise about the origin, graph the new figure on the coordinate plane, and complete each blank below with the appropriate coordinates.

A (_____,_____) → A'(_____,_____)

B (_____,_____) → B'(_____,_____)

C (_____,_____) → C'(_____,_____)

Part B: Rotate figure ABC $180°$ clockwise about the origin, graph the new figure on the coordinate plane, and complete each blank below with the appropriate coordinates.

A (_____,_____) → A'(_____,_____)

B (_____,_____) → B'(_____,_____)

C (_____,_____) → C'(_____,_____)

6. $\overline{LM}$ has endpoints $L(-7,4)$ and $M(-13,-6)$. Consider the transformation $(x,y) \rightarrow (y,-x)$ for $\overline{LM}$.

Part A: What kind of transformation is this?

Part B: What are the coordinates of $\overline{L'M'}$?

7. Point $D(-4,9)$ is rotated $90°$ counterclockwise. Which of the following is the y-coordinate of D'?

Ⓐ -9
Ⓑ -4
Ⓒ 4
Ⓓ 9

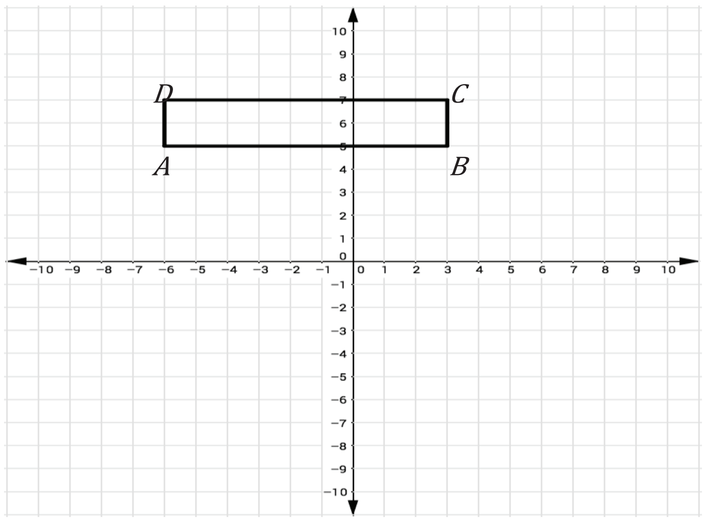
8. $\overline{EF}$ is located at $E(x,y)$ and $F(7,-10)$ and was rotated around the origin. $\overline{EF}$ became $\overline{E'F'}$ with coordinates $E'(4,5)$ and $F'(10,7)$.

Part A: What type of rotation was performed?

Part B: What are the values of x and y?

Section 3 – Topic 7
Rotations of Polygons – Part 1

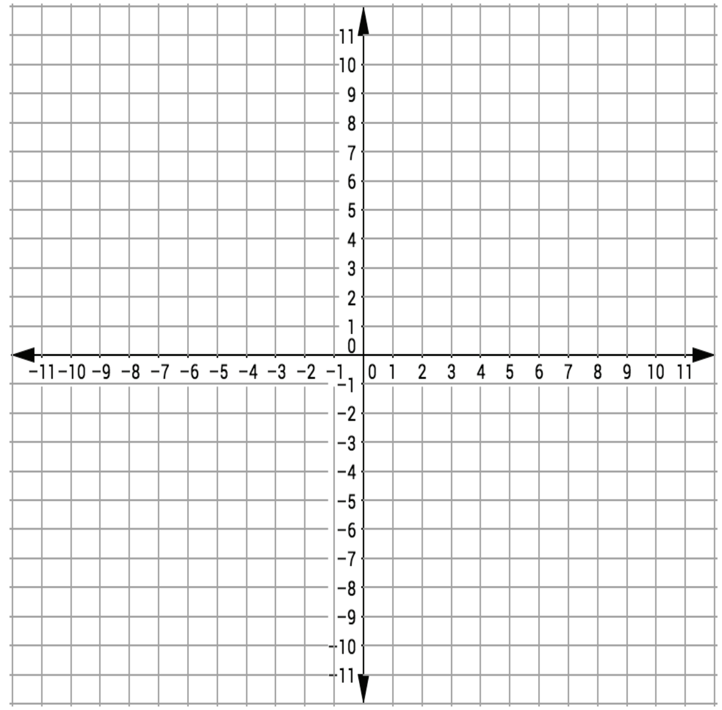
1. Consider polygon $ABCD$ shown below.



Suppose this polygon is rotated $90°, 180°, 270°$, and $360°$ clockwise about the origin. Complete the following table.

Vertices of $ABCD$	Vertices of $90°$ rotation	Vertices of $180°$ rotation	Vertices of $270°$ rotation	Vertices of $360°$ rotation
$(-6, 5)$				
$(3, 5)$				
$(3, 7)$				
$(-6, 7)$				

2. Consider the following coordinate plane.

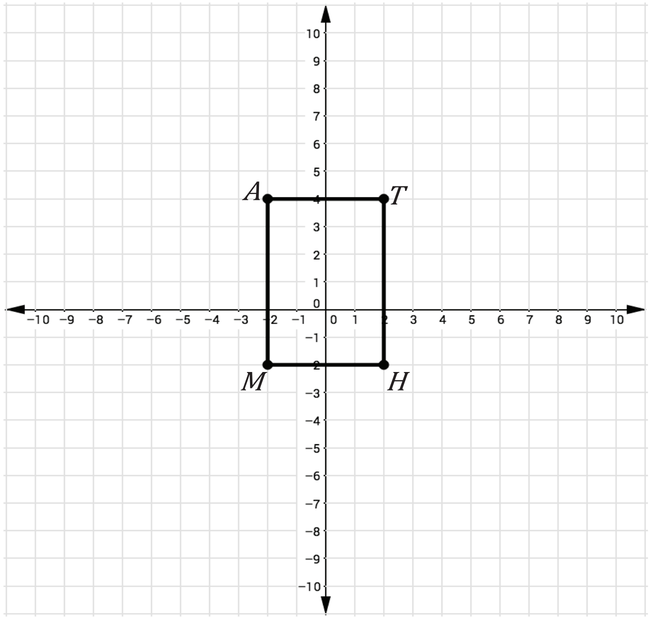


Part A: Sketch the polygon $SUMO$ with vertices at $S(10, -4), U(6, -5), M(4, 1), O(8, 2)$ on the coordinate plane above.

Part B: Rotate $SUMO$ $270°$ about the origin. What are the coordinates of $S'U'M'O'$?

Part C: Sketch the polygon S'U'M'O' on the coordinate plane above.

3. Consider Quadrilateral $MATH$ on the figure below.

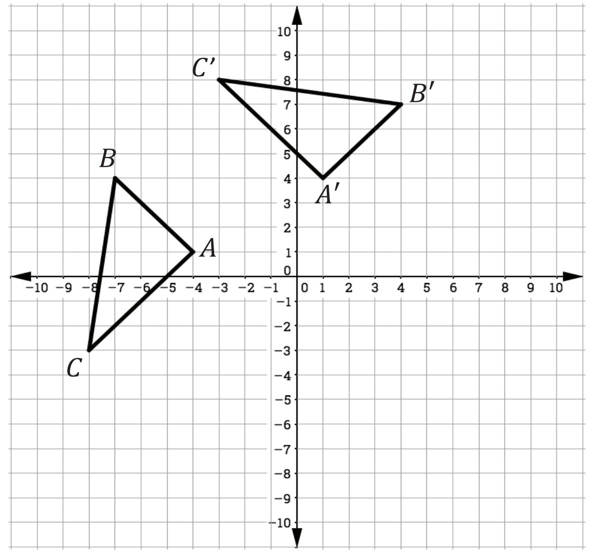


Explain why, when you rotate $MATH$ $90°$ about the origin, vertex M' shares the same location as vertex H.

4. Samuel rotated $PLUTO$ $-90°$ about the origin to generate $P'L'U'T'O'$ with vertices at $P'(2,-3), L'(-2,-5)$, $U'(-6,-1)$, $T'(-2,3)$, and $O'(1,1)$.

What is the sum of all x −coordinates of $PLUTO$?

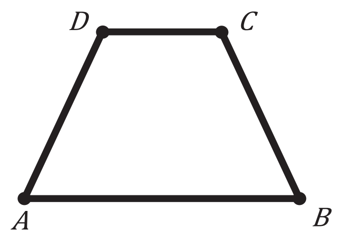
5. Sakura rotated polygon ABC $-180°$ about the origin. The following figure shows her work.



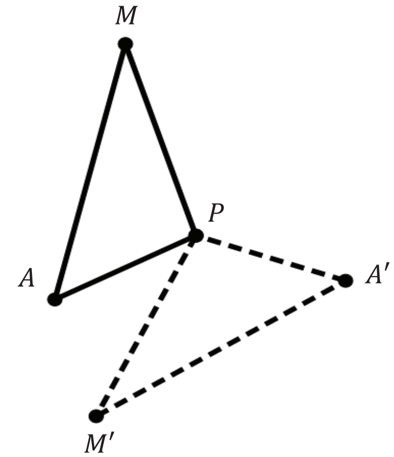
Is Sakura's work correct? Justify your answer.

Section 3 – Topic 8
Rotations of Polygons – Part 2

1. Rotate polygon $ABCD$ $135°$ about B.



2. Consider the figure below.



Part A: Determine the point of rotation. Justify your answer.

Part B: Suppose the rotation is clockwise. Determine the degrees of rotation.

3. Consider the following standard.

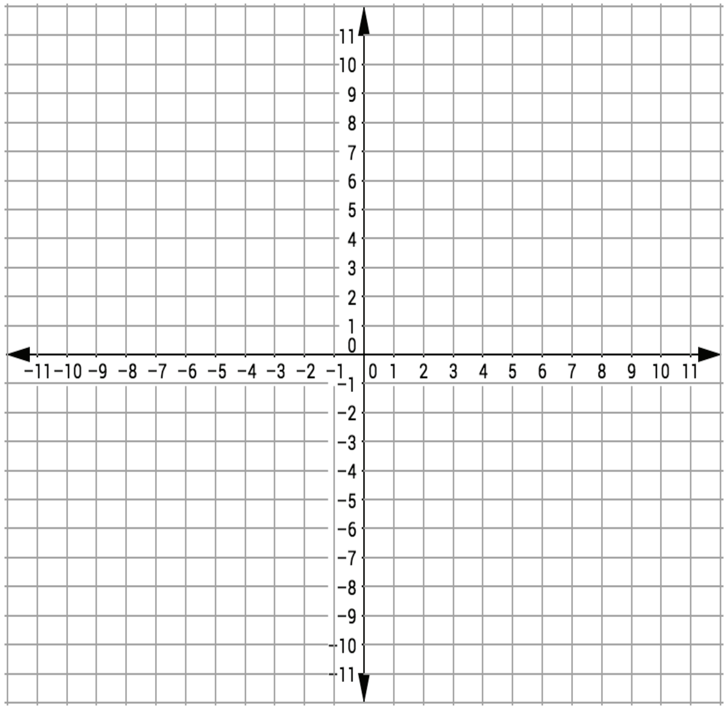
MAFS.912.G-CO.1.4.: Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

Develop a definition of a rotation in terms of any of the following: angles, circles, perpendicular lines, parallel lines, and line segments. Write your definition so that it can be used to define and perform any rotation.

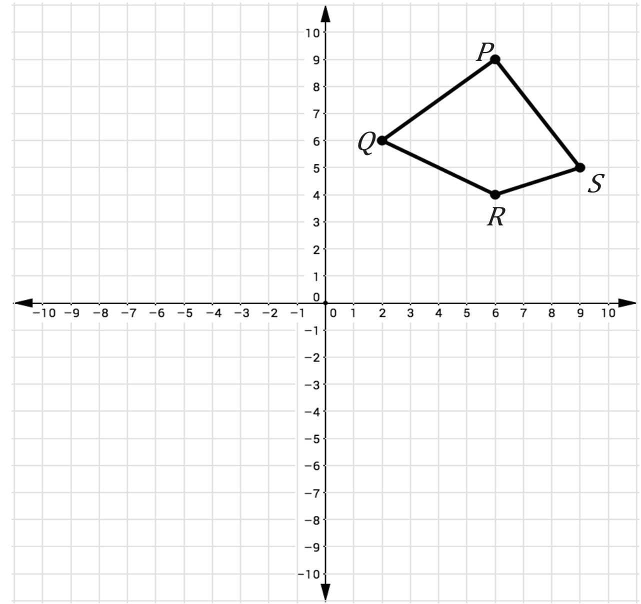
4. Consider the following statement.

Reflecting a figure twice over intersecting lines yields the same result as a rotation $180°$ about the point of intersection of the same lines.

Use the coordinate plane below to prove whether the statement is correct or incorrect.



5. Consider quadrilateral $PQRS$ on the coordinate plane below.

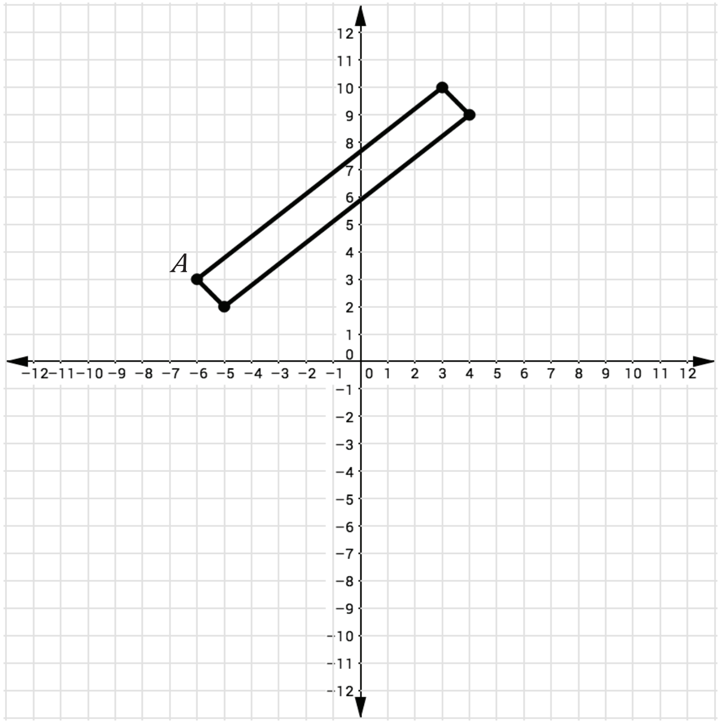


Suppose a rotation of $PQRS$ $45°$ about the origin to create $P'Q'R'S'$.

Part A: Determine which vertex or vertices will be on the second quadrant after the rotation. Justify your answer.

Part B: Consider the segment $\overline{Q'S'}$. Determine is the slope of $\overline{Q'S'}$ is positive, negative, zero or undefined.

6. Consider the quadrilateral below in which A is the only vertex labeled.

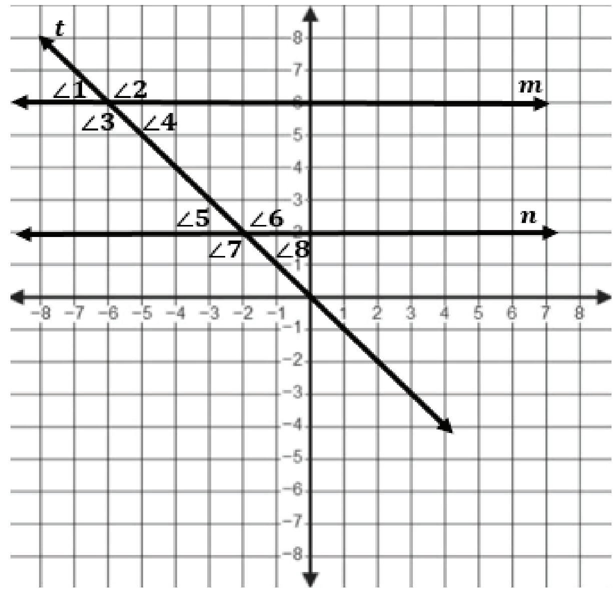


Which of the following statements are correct about the quadrilateral in the above coordinate plane? Select all that apply.

- ☐ A rotation of $180°$ around A will place A' in the fourth quadrant.
- ☐ A counterclockwise rotation between $30°$ and $140°$ around A will completely fit the quadrilateral in the second quadrant.
- ☐ A clockwise rotation of $90°$ around the origin will place A' in the first quadrant.
- ☐ A counterclockwise rotation of $270°$ around the origin will place the quadrilateral between the third and fourth quadrants.
- ☐ A rotation of $180°$ around the origin will produce the same image as a reflection across the line $y = x$.

Section 3 – Topic 9
Angle-Preserving Transformations

1. Consider the figure below in which $m \parallel n$.

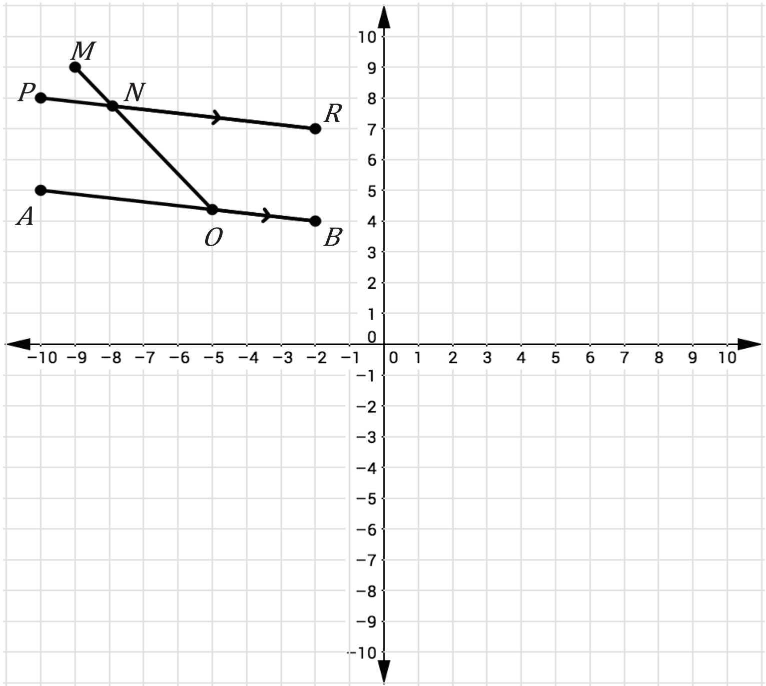


Part A: Determine the angles that are congruent with ∠6 after ∠6 has been translated 8 units to the right and 2 units down. Justify your answer.

Part B: Determine the angles that are supplementary with ∠3 after ∠3 has been rotated $180°$ clockwise.

Part C: Explain the effect that reflecting the figure across the line $y = x$ may have on each angle.

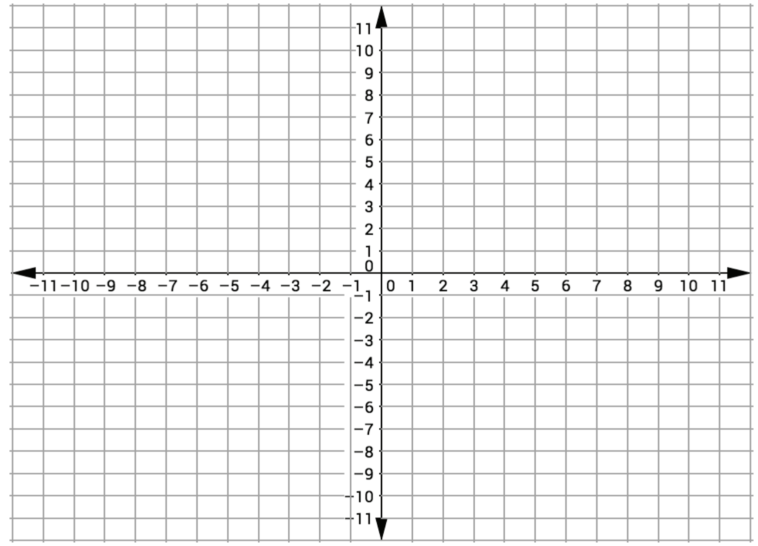
2. Consider the following figure.



Part A: Reflect the above image across the y-axis and sketch it on the coordinate plane.

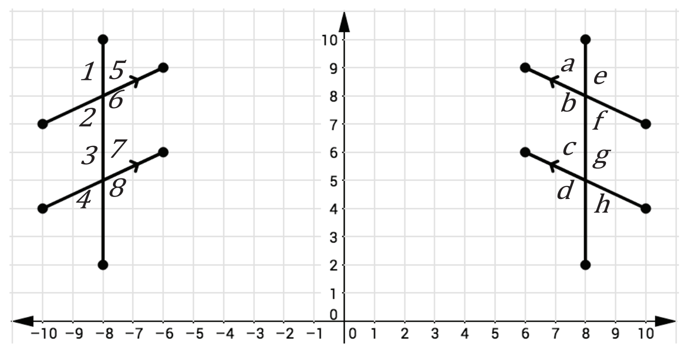
Part B: Write a paragraph proof to prove that after the reflection, $m\angle NOA = m\angle N'O'A'$.

3. Consider the transformation that you did in exercise #2 and dilate the image centered at the origin with a scale factor of $\frac{1}{2}$. Sketch the new image in the coordinate plane below.



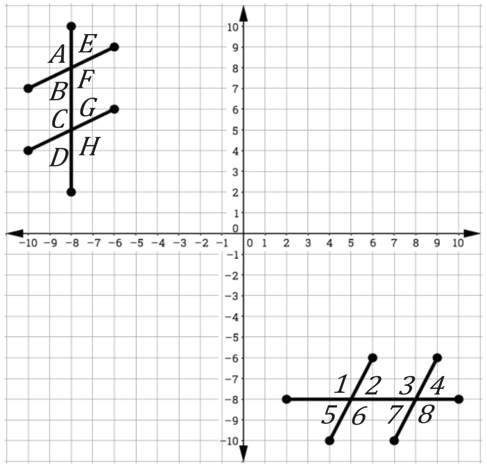
Explain the angle measures after the dilation.

4. Consider the image in Quadrant I and the pre-image in Quadrant II.



If $m\angle 4 = 5x + 14$ and $m\angle e = 7x - 2$, then $m\angle 1 =$ _______ and $m\angle c =$ ______.

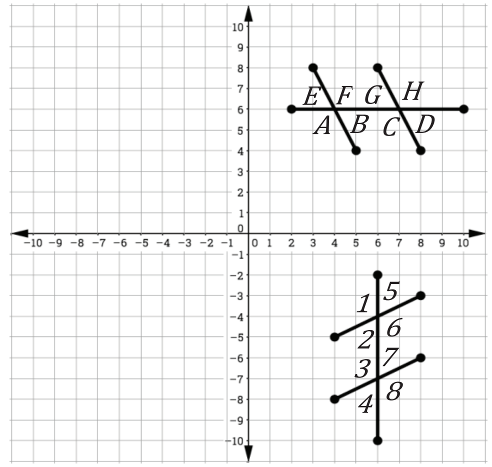
5. The figure in Quadrant IV on the coordinate plane below is a transformation of the figure in Quadrant II.



Part A: What type of transformation is shown above? Justify your answer.

Part B: Write a paragraph proof to prove that $\angle 3 \cong \angle C$ and that $\angle 3$ is a supplement angle to $\angle E$.

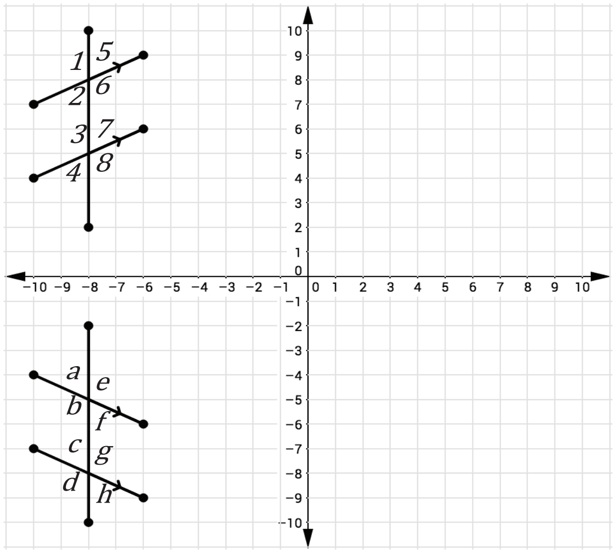
6. The figure in Quadrant I on the coordinate plane below is a transformation of the figure in Quadrant IV.



Part A: What type of transformation is shown above? Justify your answer.

Part B: Write a paragraph proof to prove that $\angle H$ and $\angle 1$ are congruent.

7. The figure in Quadrant III on the coordinate plane below is a transformation of the figure in Quadrant II.



Part A: What type of transformation is shown above? Justify your answer.

Part B: If $m\angle c = 5x - 1$, $m\angle f = 11y + 3$, $m\angle 1 = 8x - 1$, and $m\angle 8 = 17y + 9$, then determine:

$x =$

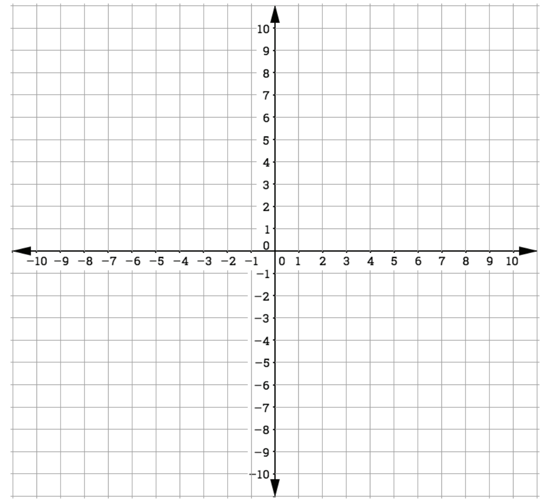
$y =$

$m\angle 6 =$

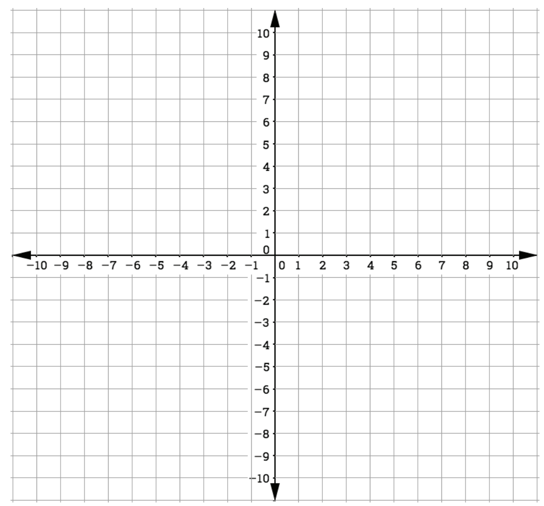
$m\angle a =$

Section 3 – Topic 10
Symmetries of Regular Polygons

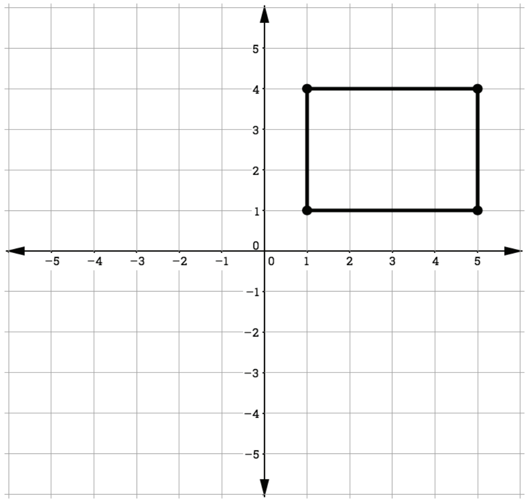
1. Sketch an example of a symmetrical polygon. Explain why it is symmetrical.



2. Sketch an example of an asymmetrical polygon. Explain why it is *not* symmetrical.



3. Consider the rectangle below.



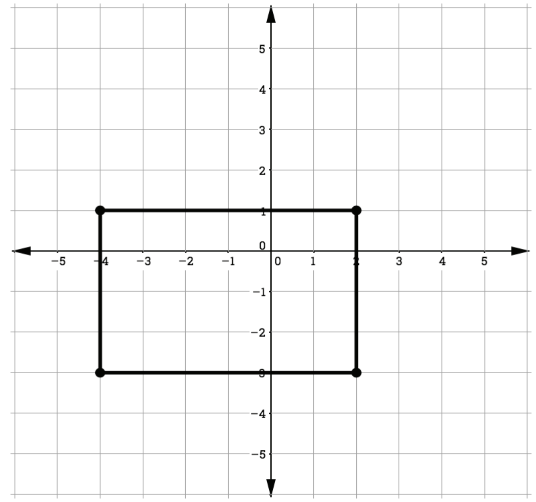
Part A: If the image is reflected across the line $x = 3$, does the transformation result in the original pre-image?

Part B: If the image is reflected across the line $y = 3$, does the transformation result in the original pre-image?

4. Complete the following paragraph.

In regular polygons, if the number of sides, n, is odd, the lines of symmetry will pass through a _______________ and the __________________ of the opposite side. If n is even, then the lines of symmetry will pass through two ______________ vertices or the _______________ of two opposite sides.

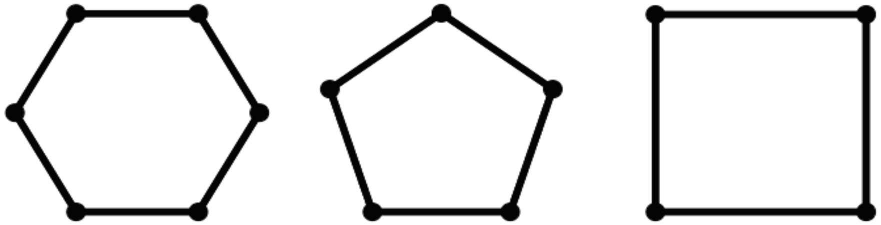
5. Consider the rectangle below.



Part A: Write the equation of a *horizontal* line that will map the figure onto itself after a reflection across that line.

Part B: Write the equation of a *vertical* line that will map the figure onto itself after a reflection across that line.

6. Draw the lines of symmetry on the regular polygons below.



7. In a regular polygon with n sides, how many lines of symmetry are possible?

Ⓐ $\frac{1}{2}n$

Ⓑ n

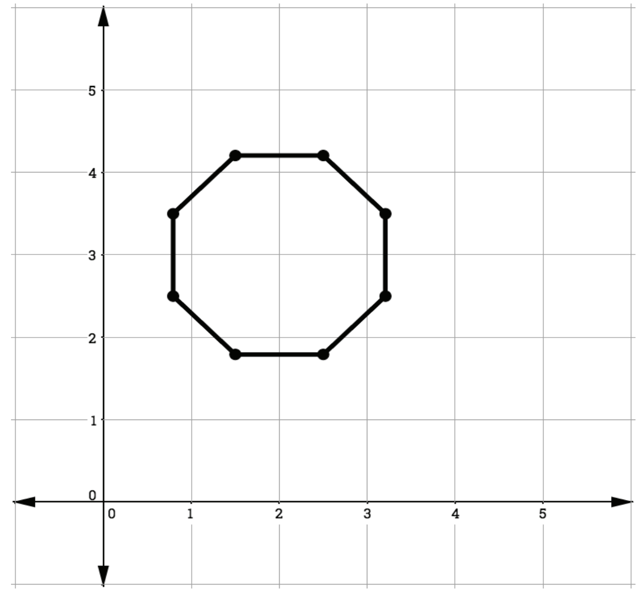
Ⓒ $2n$

Ⓓ $4n$

8. A regular pentagon can be rotated by any multiple of $\boxed{}$ about its central angle to be mapped back onto itself.

9. Consider a regular hexagon centered at $(3, 4)$. Describe a rotation that will map this regular hexagon onto itself.

10. Which of the following rotations will map the regular polygon below, centered at $(2, 3)$, onto itself? Select all that apply.



- ☐ Rotation of $360°$ centered at the origin.
- ☐ Rotation of $360°$centered at $(2, 3)$.
- ☐ Rotation of $90°$ centered at $(3, 5)$.
- ☐ Rotation of $45°$ centered at $(2, 3)$.
- ☐ Rotation of $225°$ centered at the origin.
- ☐ Rotation of $270°$ centered at $(2, 3)$.
- ☐ Rotation of $181°$ centered at $(2, 3)$

Section 4: Non-Rigid Transformations, Congruence, and Similarity
Student Learning Plan

Topic Number	Topic Name	Date Completed	Study Expert(s)	Check Your Understanding Score
1	Examining and Using Dilations – Part 1			
2	Examining and Using Dilations – Part 2			
3	Dilation of Polygons			
4	Compositions of Transformations of Polygons – Part 1			
5	Compositions of Transformations of Polygons – Part 2			
6	Congruence of Polygons			
7	Similarity of Polygons			

What did you learn in this section? What questions do you still have?

Who was your favorite Study Expert for this section? Why?

Section 4 – Topic 1
Examining and Using Dilations – Part 1

1. What is the difference between an image under a dilation centered at the origin under a of scale factor $0 < k < 1$ and an image under a scale factor $k > 1$?

2. What happens when an image is dilated using the following scale factors with the center at the origin?

   $k = 1$

   $0 < k < 1$

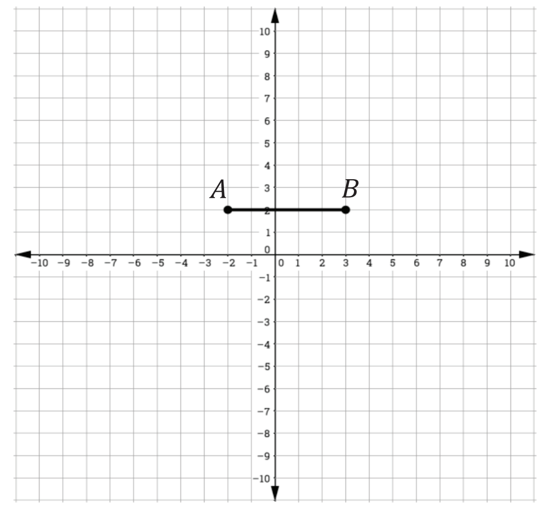
   $k > 1$

   $k = 0$

3. Describe the dilation of a line segment by a scale factor of $\frac{1}{2}$ centered at the origin.

4. Consider $\overline{GK}$ located at $G(0, 0)$ and $K(4, 4)$. Describe the dilation of $\overline{GK}$ by a scale factor of 2 centered at $(-2, -4)$.

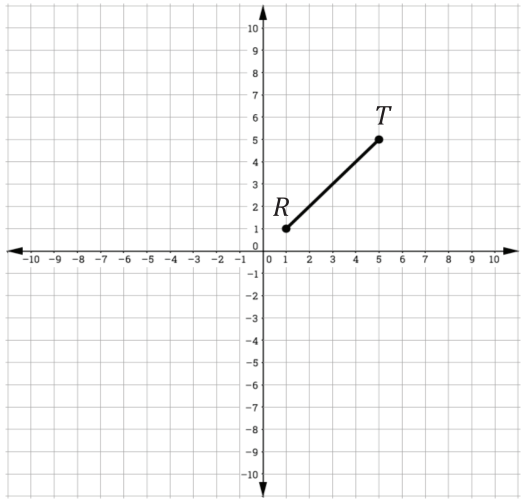
5. Consider the following graph.



Part A: Under a dilation of scale factor 3 centered at the origin, $\overline{AB}$ becomes $\overline{A'B'}$. Determine the coordinates of $\overline{A'B'}$ and sketch $\overline{A'B'}$ on the graph.

Part B: Under a dilation of scale factor $\frac{1}{2}$ centered at the origin, $\overline{AB}$ becomes $\overline{DC}$. Determine the coordinates of $\overline{DC}$ and sketch $\overline{DC}$ on the graph.

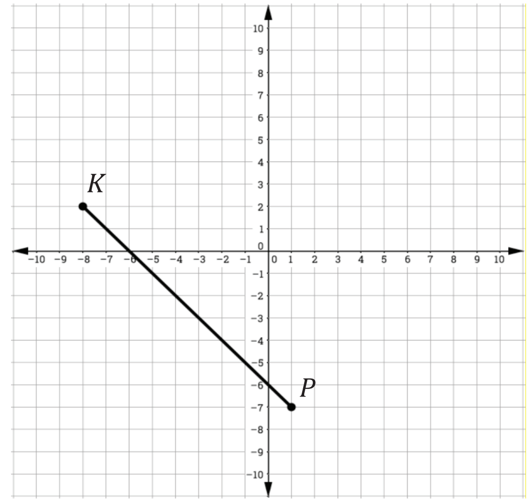
6. Consider the following graph.



Part A: Under a dilation of scale factor $\frac{3}{2}$ centered at $(3, 7)$, $\overline{RT}$ becomes $\overline{R'T'}$. Determine the coordinates of $\overline{R'T'}$ and sketch $\overline{R'T'}$ on the graph.

Part B: Under a dilation of scale factor $\frac{1}{2}$ centered at $(-1, 9)$, $\overline{RT}$ becomes $\overline{LM}$. Determine the coordinates of $\overline{LM}$ and sketch $\overline{LM}$ on the graph.

7. Consider the following graph.



Under a dilation of scale factor $\frac{2}{3}$ centered at $(-8, 2)$, $\overline{KP}$ becomes $\overline{K'P'}$. Determine the coordinates of $\overline{K'P'}$ and sketch $\overline{K'P'}$ on the graph.

Section 4 – Topic 2
Examining and Using Dilations – Part 2

1. Suppose $\overline{WZ}$ is on a coordinate plane located at $W(-3,5)$ and $Z(1,1)$. Under a dilation centered at the origin, $\overline{WZ}$ becomes $\overline{W'Z'}$ with coordinates $W'(-12,20)$ and $Z'(4,4)$. What is the scale factor for this dilation?

2. Suppose $\overline{HJ}$ is on a coordinate plane located at $H(-5,2)$ and $J(1,4)$. Under a dilation centered at $(3,2)$, $\overline{HJ}$ becomes $\overline{H'J'}$ with coordinates $H'(-1,2)$ and $J'(2,3)$. What is the scale factor for this dilation?

3. Suppose $\overline{TU}$ is on a coordinate plane located at $T(-6,-2)$ and $U(2,-6)$. Under a dilation of scale factor $\frac{1}{4}$, $\overline{TU}$ becomes $\overline{T'U'}$ with coordinates $T'\left(-\frac{3}{2},-\frac{1}{2}\right)$ and $U'\left(\frac{1}{2},-\frac{3}{2}\right)$. Where is the center of dilation located?

   Ⓐ $(0,0)$
   Ⓑ $(2,2)$
   Ⓒ $(4,0)$
   Ⓓ $(4,4)$

4. Line r is mapped onto the line m by a dilation centered at the origin with a scale factor of 4. The equation of line r is $6x-3y=12$. What is the equation for line m?

5. $\overline{AB}$ has coordinates $A(-15,5)$ and $B(0,-10)$. Find the coordinates of $\overline{A'B'}$ after a dilation with a scale factor of $\frac{1}{5}$ centered at the origin.

6. $\overline{SV}$ has coordinates $S(-6,1)$ and $V(-1,-7)$. Find the coordinates of $\overline{S'V'}$ after a dilation with a scale factor of 3 centered at $(-1,1)$.

7. $\overline{M'T'}$ has coordinates $M'(-1,3)$ and $T'(12,-6)$, and it is the result of the dilation of $\overline{MT}$ centered at the origin. The coordinates of $\overline{MT}$ are $M\left(-\frac{1}{3},1\right)$ and $T(4,-2)$. Complete the following algebraic description so that it represents the transformation of $\overline{MT}$.

   $(x,y)\rightarrow(\boxed{}\,x,\ \boxed{}\,y)$

8. $\overline{CD}$ has coordinates $C(-8,-2)$ and $D(-4,-12)$. Determine the coordinates of $\overline{C'D'}$ if $(x,y)\rightarrow\left(\frac{1}{2}x,\frac{1}{2}y\right)$ and describe the transformation from $\overline{CD}$ to $\overline{C'D'}$.

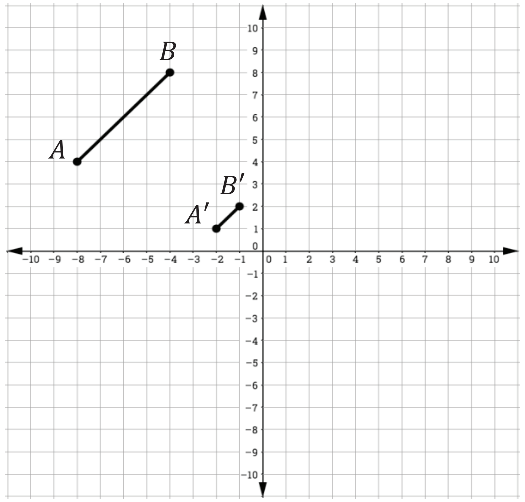
9. Suppose the line l represented by $f(x)=x+3$ is transformed into $g(x)=2\big(f(x-1)\big)+3$.

   Part A: Describe the transformation from $f(x)$ to $g(x)$.

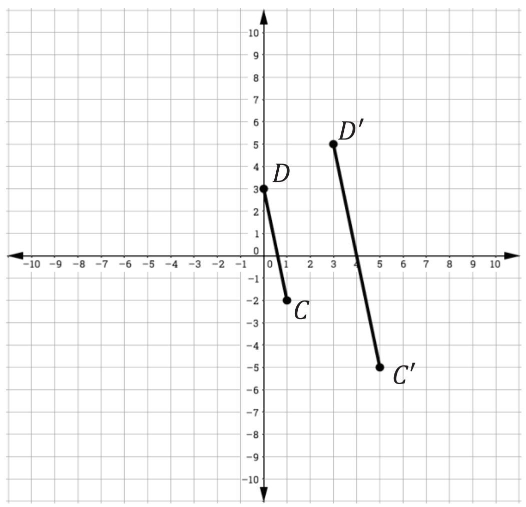
   Part B: What is the y-coordinate of $g(0)$?

10. Consider the following graphs and describe the transformations illustrated in each graph.

Graph A:



Graph B:



Section 4 – Topic 3
Dilation of Polygons

1. Hexagon $FUTBOL$ has coordinates $F(3,6)$, $U(5,2)$, $T(2,-3)$, $B(-3,-2)$, $O(-5,1)$, and $L(-2,5)$, and it is dilated by a scale factor of five centered at the origin.

   What are the coordinates of $F'U'T'B'O'L'$?

2. Pentagon $CRAFT$ has coordinates $C(-2,1)$, $R(-5,2)$, $A(-6,7)$, $F(-3,9)$, and $T(-1,6)$ and it is dilated by a scale factor of $\frac{2}{3}$ centered at the origin.

   Part A: Determine in which quadrant $C'R'A'F'T'$ will be located.

   Part B: Let the origin be point O. Determine the difference in length between $\overline{OC}$ and $\overline{OC'}$.

   Part C: What is the x-coordinate of A'?

   Part D: What is the difference between the y-coordinate of F' and the y-coordinate of T'?

3. Consider the following statement.

   A dilation of square $ABCD$ centered at the origin with a scale factor of $\frac{4}{3}$ will place $A'B'C'D'$ closer to point $(0,0)$.

   Determine the validity of this statement. Justify your answer.

4. Quadrilateral $CART$ is dilated with the center at the origin and a scale factor of $\frac{6}{5}$.

Describe Quadrilateral $CART$ and Quadrilateral $C'A'R'T'$ by completing the table below with the most appropriate answer.

Quadrilateral $CART$	**Quadrilateral $C'A'R'T'$**
(x, y)	(,)
$C(-5, 15)$	C'(,)
A(,)	$A'(12, 6)$
R(,)	$R'(-24, -12)$
$T(-25, 0)$	T'(,)

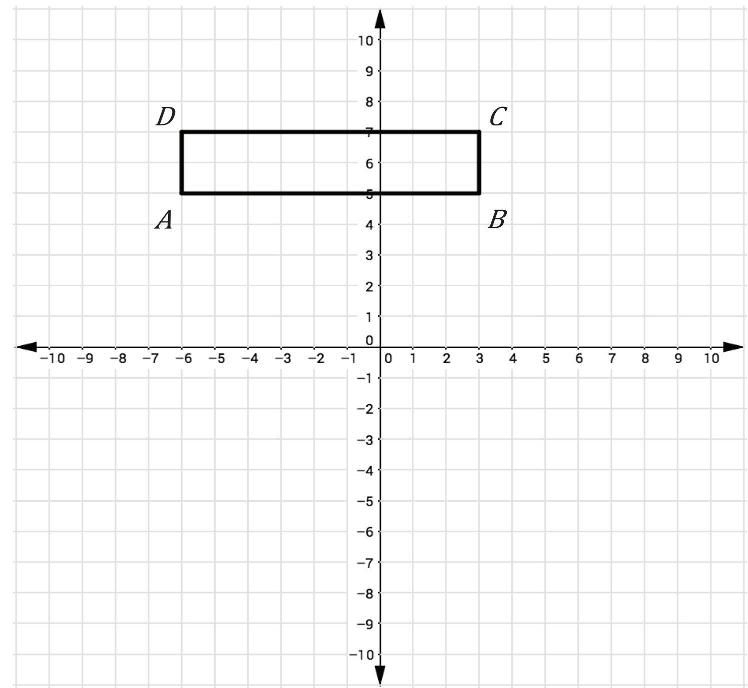
5. Consider the following standard.

MAFS.912.G-CO.1.4.: Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

Part A: Determine what dilations have in common with translations, reflections and rotations in terms of any of the following: angles, circles, perpendicular lines, parallel lines, and line segments.

Part B: Develop a comparison between dilations and translations, reflections and rotations in terms of any of the following: angles, circles, perpendicular lines, parallel lines, and line segments.

6. Consider rectangle $ABCD$.



Dilate $ABCD$ by a scale factor of $\frac{1}{2}$ using a center of dilation of $(-2, 2)$. Draw $A'B'C'D'$ on the same coordinate plane.

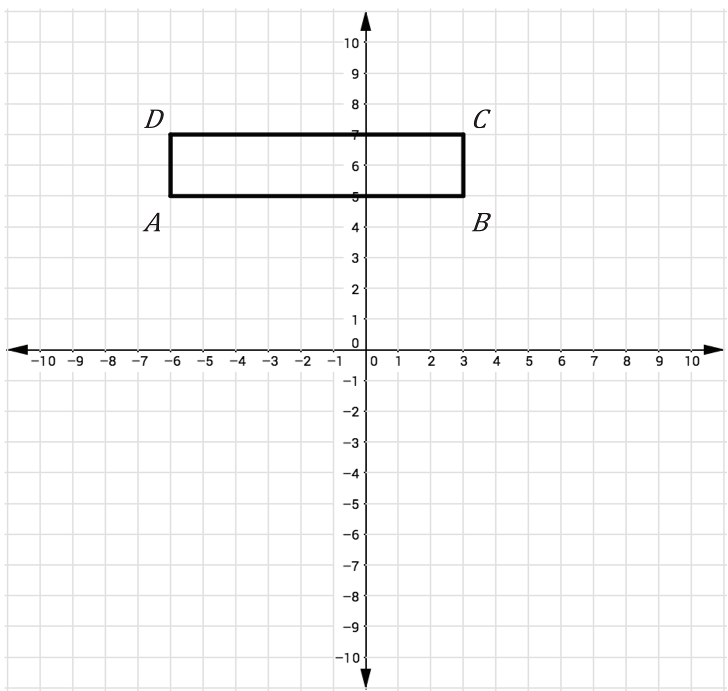
7. Triangle FIP was dilated by a scale factor of $\frac{1}{4}$ centered at the origin to create triangle $F'I'P'$, which has coordinates $F'\left(1,-\frac{3}{2}\right), I'(2,-1), P'(1,2)$. Write the coordinates of the vertices of triangle FIP in the spaces provided below.

$F($______, ______$)$

$I($______, ______$)$

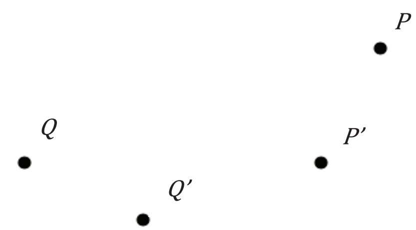
$P($______, ______$)$

8. Consider rectangle $ABCD$.



Suppose $ABCD$ is dilated with the center at vertex C and a scale factor of three. Determine the sum of the y-coordinates of $A'B'C'D'$.

9. Consider the diagram below.



Find the center of dilation and the scale factor that takes P to P' and Q to Q', if a dilation exists.

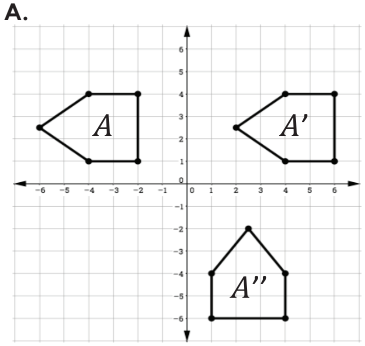
Section 4 – Topic 4
Compositions of Transformations of Polygons – Part 1

1. How is the movement of a basketball during a game a real-life example of a Composition of Transformations?

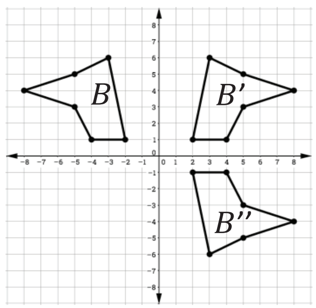
2. Next to each composition of transformations, write the letter for the corresponding graph from the options below.

	Glide Reflection		Composition of isometries (translation, then rotation)
	Double Reflection		Composition of isometries (reflection, then translation)

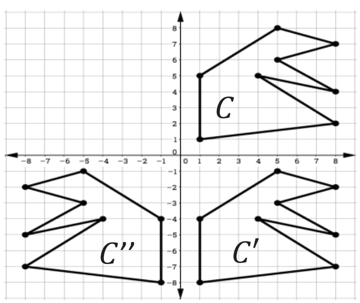
A.



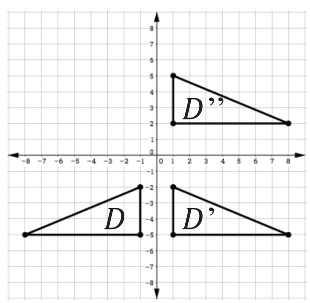
B.



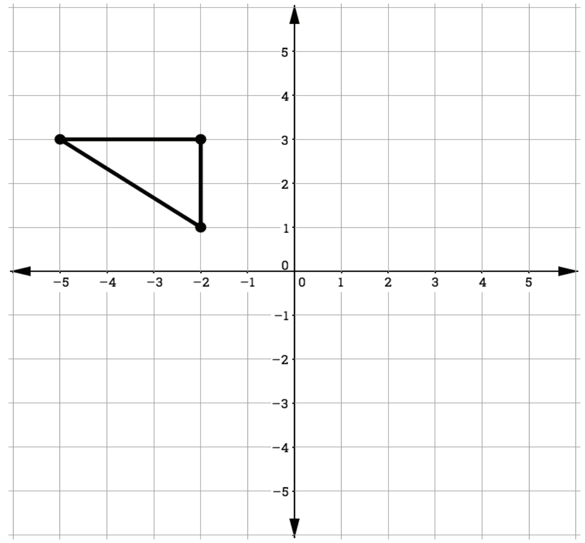
C.



D.



3. Consider the figure below and represent a composition of isometries by reflecting the figure over the x-axis and then translating the resulting image following the function rule $(x, y) \rightarrow (x + 1, y - 2)$.



4. Which of the following is ***not*** a composition of isometries?

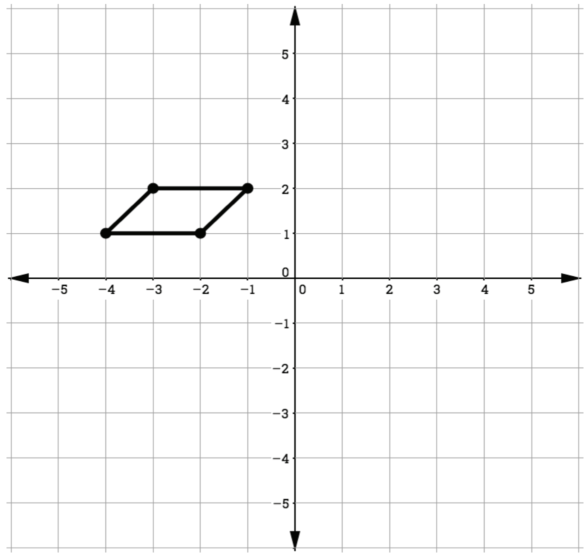
Ⓐ Reflection over $x = 2$, then rotation $90°$ clockwise about the origin.

Ⓑ Dilation with scale factor $\frac{1}{2}$, then rotation $270°$ clockwise about the origin.

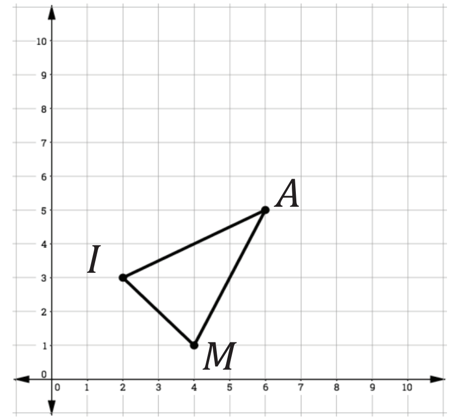
Ⓒ Translation $(x, y) \rightarrow (x - 2, y + 1)$, then reflection over the x-axis.

Ⓓ Reflection over the x-axis, then reflection over the y-axis.

5. Dilate the figure below with a scale factor of 1.5 centered at the origin and then rotate the figure $90°$ clockwise about the origin.



6. Triangle MIA is shown.



There are four highlights in the paragraph that show equations or phrases that are missing. For each highlight, write the correct equation or phrase.

The vertices of $\triangle MIA$ are $M(4,1)$, $I(2,3)$, and $A(6,5)$. A reflection across the line ________________ and then across the line ________________ is the same as a rotation of ________________ clockwise about the origin because the lines ____.

Section 4 – Topic 5
Compositions of Transformations of Polygons – Part 2

1. Consider the polygon $GEOM$ with coordinate $G(0,-2)$, $E(-1,2)$, $O(-5,1)$, $M(-5,-6)$. If we rotate $GEOM$ $90°$ clockwise about the origin, then reflect over the y-axis, what are the coordinates of $G''E''O''M''$?

$G(0,-2) \rightarrow G'(\_\_\_\_\_,\_\_\_\_\_) \rightarrow G''(\_\_\_\_\_,\_\_\_\_\_)$

$E(-1,2) \rightarrow E'(\_\_\_\_\_,\_\_\_\_\_) \rightarrow E''(\_\_\_\_\_,\_\_\_\_\_)$

$O(-5,1) \rightarrow O'(\_\_\_\_\_,\_\_\_\_\_) \rightarrow O''(\_\_\_\_\_,\_\_\_\_\_)$

$M(-5,-6) \rightarrow M'(\_\_\_\_\_,\_\_\_\_\_) \rightarrow M''(\_\_\_\_\_,\_\_\_\_\_)$

2. Consider polygon $ABCD$ with coordinates $A(8,12), B(10,4), C(4,4), D(2,8)$.

Part A: What are the coordinates of $A"B"C"D"$ if we reflect it over the line $y = 2$, then dilate by a scale factor of $\frac{1}{2}$ centered at the origin?

$A(8,12) \rightarrow A'(\_\_\_\_\_,\_\_\_\_\_) \rightarrow A''(\_\_\_\_\_,\_\_\_\_\_)$

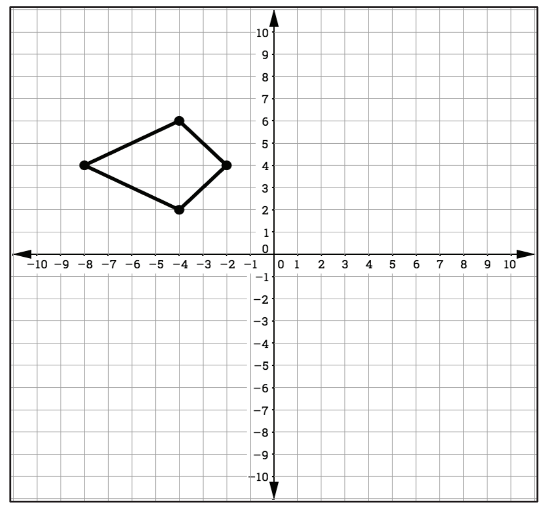
$B(10,4) \rightarrow B'(\_\_\_\_\_,\_\_\_\_\_) \rightarrow B''(\_\_\_\_\_,\_\_\_\_\_)$

$C(4,4) \rightarrow C'(\_\_\_\_\_,\_\_\_\_\_) \rightarrow C''(\_\_\_\_\_,\_\_\_\_\_)$

$D(2,8) \rightarrow D'(\_\_\_\_\_,\_\_\_\_\_) \rightarrow D''(\_\_\_\_\_,\_\_\_\_\_)$

Part B: If polygon $A"B"C"D"$ is now reflected back over the line $y = 2$, what are the coordinates of the new polygon $A'''B'''C'''D'''$? Justify your answer.

3. The polygon below is rotated 270° counterclockwise about the origin, then reflected over the x-axis. What transformation will map the polygon back to its original image?



4. The polygon $WXYZ$ is translated by $(x+1, y+5)$, then reflected over the line $y = x$. The resulting polygon $W''X''Y''Z''$ has coordinates $W''(-4,5)$, $X''(-1,5)$, $Y''(-1,1)$, $Z''(-4,1)$. What are the coordinates of the original polygon $WXYZ$?

Section 4 – Topic 6
Congruence of Polygons

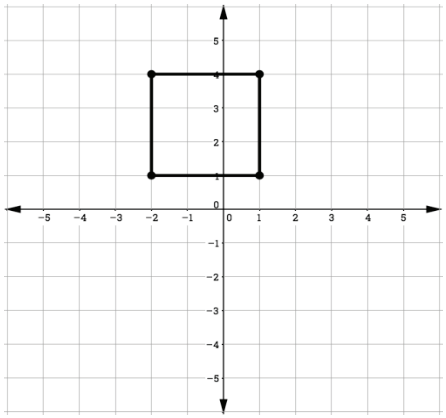
1. Consider the polygons below.

Polygon 1 Polygon 2 Polygon 3 Polygon 4

Determine if any of the following listed pairs are congruent. Select all that apply.

- ☐ Polygon 1 and Polygon 2
- ☐ Polygon 1 and Polygon 3
- ☐ Polygon 1 and Polygon 4
- ☐ Corresponding sides of Polygon 1 and sides of Polygon 2
- ☐ Corresponding sides of Polygon 1 and sides of Polygon 4
- ☐ Corresponding angles of Polygon 1 and Polygon 3
- ☐ Corresponding angles of Polygon 1 and Polygon 4

2. Which of the following sets of coordinates represents a square that is congruent to the one below?



Ⓐ $S(1,1), T(1,5), O(-2,1), P(-2,5)$

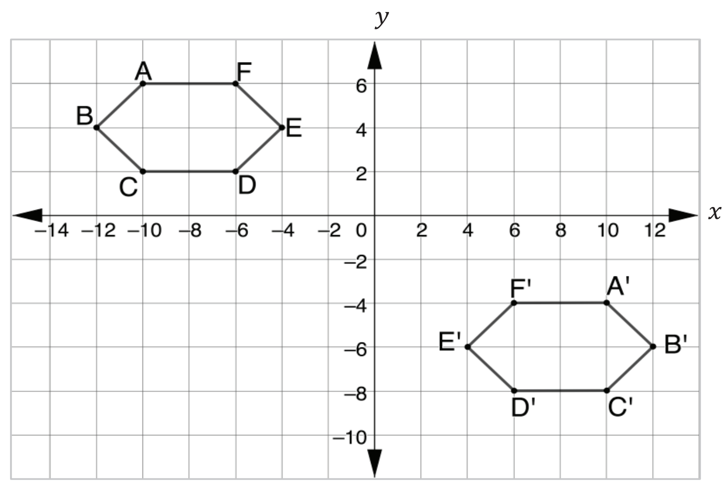
Ⓑ $E(6,-2), F(6,3), G(1,3), H(1,-2)$

Ⓒ $F(2,-1), L(5,-1), A(2,-4), T(5,-4)$

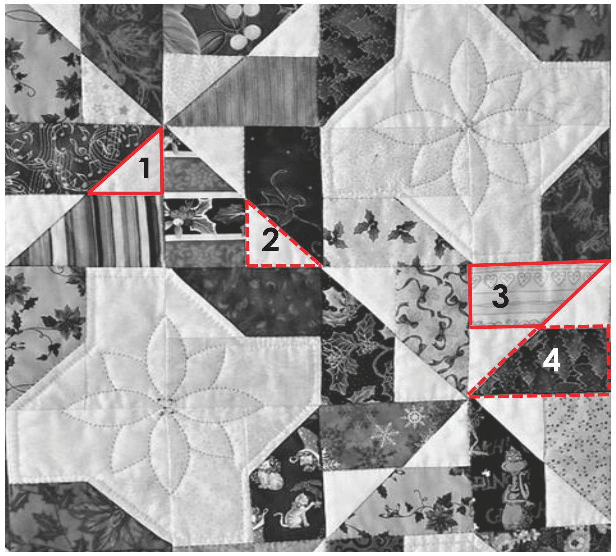
Ⓓ $G(1,1), E(2,4), M(5,4), S(5,1)$

3. Quadrilateral $HALK$ is congruent to quadrilateral $FORT$. $m\angle H = 60°$, $m\angle L = 152°$, and $m\angle T = 42°$. What is $m\angle A$?

4. Titus is asked to prove hexagon $FEDCBA$ is congruent to hexagon $F'E'D'C'B'A'$ in the graph below. Titus thinks that if he transforms hexagon $FEDCBA$ by $(x,y) \rightarrow (x+16, y-10)$ he can show the two figures are congruent. Is he correct? Explain why or why not. If Titus is incorrect, what series of transformations will correctly prove $FEDCBA \cong F'E'D'C'B'A'$?



5. The quilt shown below is made up of polygons that are arranged to create a pattern.

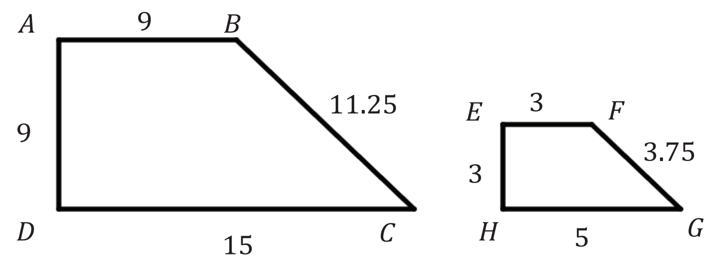


Part A: Show that $\Delta 1$ and $\Delta 2$ are congruent by describing a series of transformations that carry $\Delta 1$ onto $\Delta 2$.

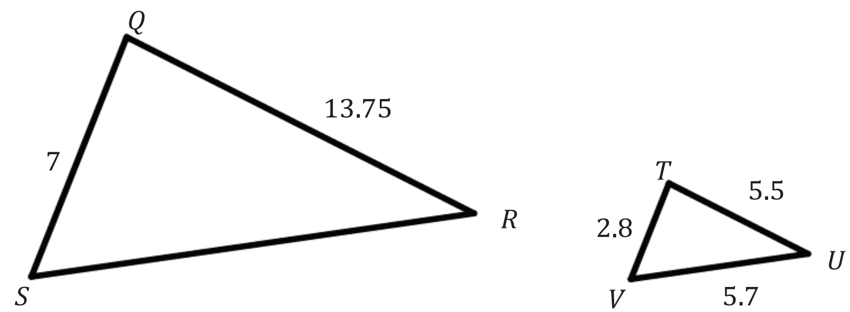
Part B: Show that trapezoid 3 and trapezoid 4 are congruent by describing a series of transformations that carry trapezoid 3 onto trapezoid 4.

Section 4 – Topic 7
Similarity of Polygons

1. Figure $ABCD$ is the preimage of figure $EFGH$. The two figures are similar. What is the scale factor?



2. Consider similar figures QRS and TUV below where QRS is the preimage of TUV.

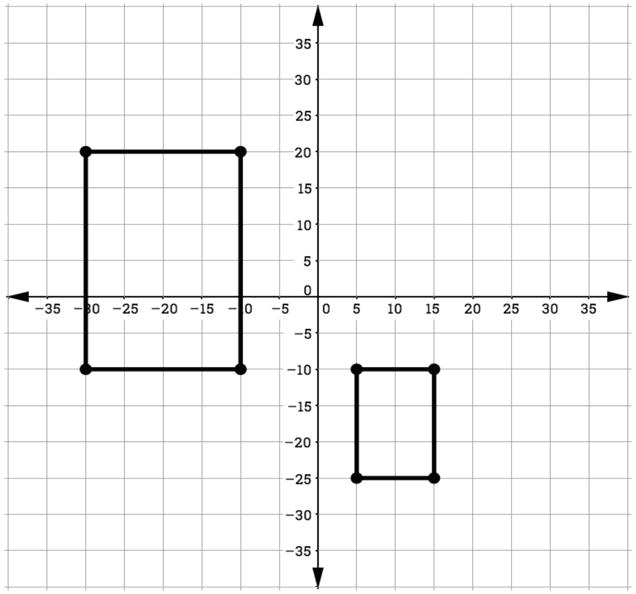


Part A: What is the scale factor?

Part B: Find the length of $\overline{RS}$.

3. Compare and contrast similar and congruent figures.

4. Are the rectangles below similar? Justify your answer.



5. Triangle GAP is similar to triangle DOT. $\overline{GA}$ is 4.4 inches long, $\overline{AP}$ is 8.3 inches long, $\overline{OT}$ is 24.9 inches long, and $\overline{TD}$ is 15.6 inches long. How long is $\overline{GP}$?

6. You are printing posters for a concert and need them to be 35” tall. The small copy you have is 8” wide by 10” inches tall.

Part A: What scale factor should you use when enlarging the image to make sure proper proportions are maintained?

Part B: How wide will the enlarged poster be?

Part C: How do the areas of the posters relate to one another? What are two ways you can justify your answer?

7. John is building a soccer table and wants it to be proportional to a real soccer field. A real soccer field is 336 feet long and 210 feet wide. What will be the area of his soccer table if he wants it to be 2.5 feet wide?

8. Which transformation would result in the area of a polygon being different from the area of its pre-image?

Ⓐ Dilated by a scale factor of 1
Ⓑ Dilated by a scale factor of 2.5
Ⓒ Rotated 270° counterclockwise about the origin
Ⓓ Translated four units up and two units to the right

9. Which of the following would *not* help justify that two shapes are similar?

Ⓐ They are the same shape.
Ⓑ They have the same number of vertices.
Ⓒ One set of corresponding sides is congruent, but another is not.
Ⓓ All sets of corresponding sides are in proportion to each other.

Section 5: Triangles – Part 1
Student Learning Plan

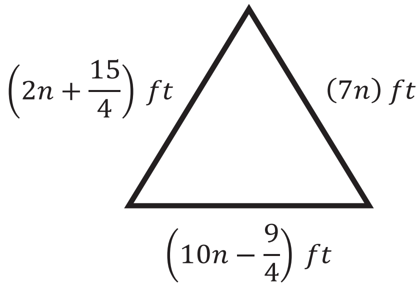
Topic Number	Topic Name	Date Completed	Study Expert(s)	Check Your Understanding Score
1	Introduction to Triangles – Part 1			
2	Introduction to Triangles – Part 2			
3	Triangles in the Coordinate Plane			
4	Triangle Congruence – SSS and SAS – Part 1			
5	Triangle Congruence – SSS and SAS – Part 2			
6	Triangle Congruence – ASA and AAS – Part 1			
7	Triangle Congruence – ASA and AAS – Part 2			
8	Base Angle of Isosceles Triangles			
9	Using the Definition of Triangle Congruence in Terms of Rigid Motions			
10	Using Triangle Congruency to Find Missing Variables			

What did you learn in this section? What questions do you still have?

Who was your favorite Study Expert for this section? Why?

Section 5 – Topic 1
Introduction to Triangles – Part 1

1. Consider the diagram below of an equilateral triangle.

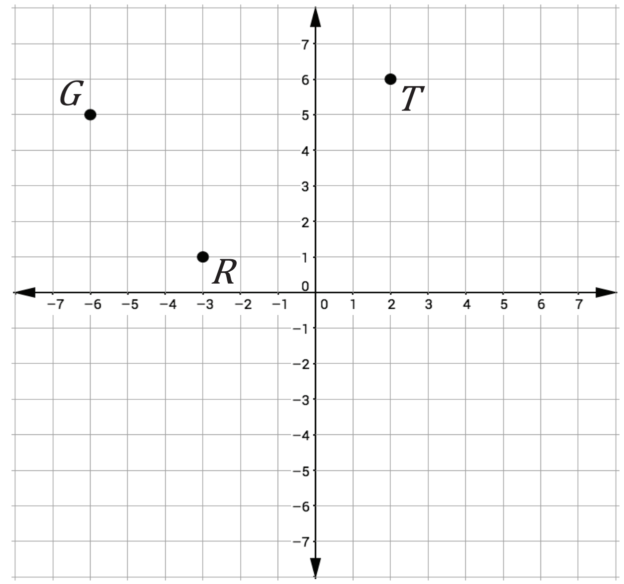


How long is each side of the triangle? Justify your answer.

2. Match the description to the type of triangle that is produced.

	Description	Type of Triangle
________	One Obtuse Angle	**A.** Equilateral
________	All 60° angles	**B.** Acute
________	No Congruent sides	**C.** Obtuse
________	One Right Angle	**D.** Equiangular
________	Three Congruent Sides	**E.** Isosceles
________	Three Acute Angles	**F.** Scalene
________	Two Congruent Sides	**G.** Right

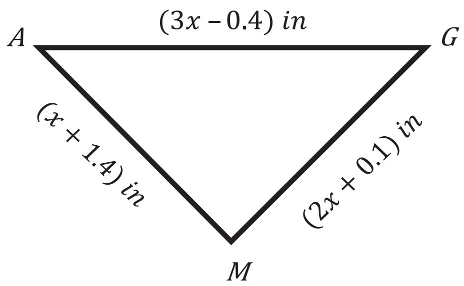
3. Consider the figure below.



Part A: Mrs. Konsdorf claims that angle R is a right angle. Is Mrs. Konsdorf correct? Explain your reasoning.

Part B: If T is transformed under the rule $(x, y) \rightarrow (x-1, y-2)$, then does T' form a right angle at $\angle GRT'$?

4. Consider the triangle below.



Part A: If ΔAMG is an isosceles triangle with base $\overline{AG}$, what is the value of x? Justify your answer.

Part B: What is the length of each leg?

Part C: What is the length of the base?

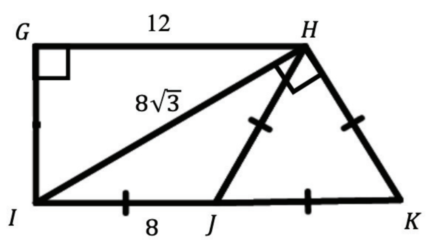
5. Consider the diagram on the right. Classify each triangle as equilateral, isosceles, or scalene.

ΔIHG: ____________________________

ΔHJI: ____________________________

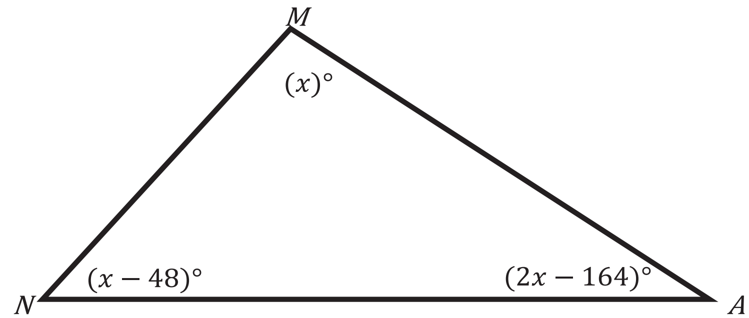
ΔKHI: ____________________________

ΔHJK: ____________________________



Section 5 – Topic 2
Introduction to Triangles – Part 2

1. Consider the figure below.



Determine the measure of each interior angle of $\triangle MAN$ and classify the triangle.

$m\angle A =$

$m\angle M =$

$m\angle N =$

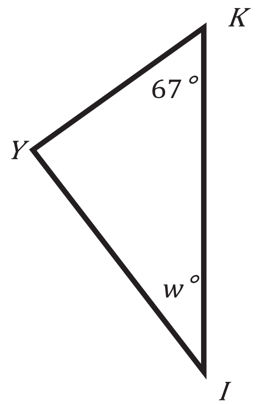
ΔMAN is a(n) __________________________ triangle.

2. Triangle CAT has vertices at $C(-6, 0)$, $A(4, -2)$, and $T(5, 3)$.

What type of triangle is CAT?

Ⓐ Obtuse
Ⓑ Isosceles
Ⓒ Equilateral
Ⓓ Right

3. *Garden Plus LLC* is fencing a triangular garden (pictured below) for Mr. Gold.



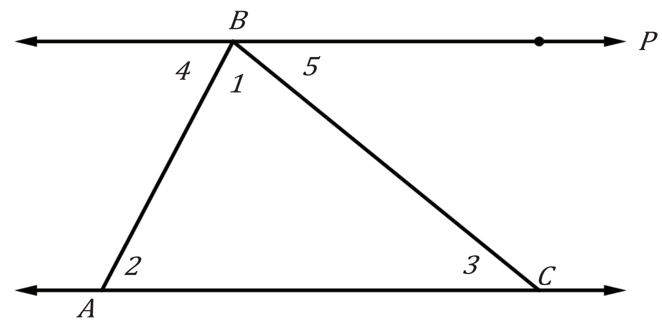
Part A: Determine the expression for the measure of angle Y.

Part B: If $m\angle I = 33$ and $m\angle Y = 14d - 19$, then determine the value of d.

4. Triangle OMG has vertices at $O(4, -2)$, $M(5, 3)$, and $G(-6, 0)$. If point G is transformed under the translation of $(x, y) \rightarrow (x + 3, y + 2)$, then $\triangle OMG'$ is

- O equilateral.
- O isosceles.
- O scalene.

5. Consider the following figure.

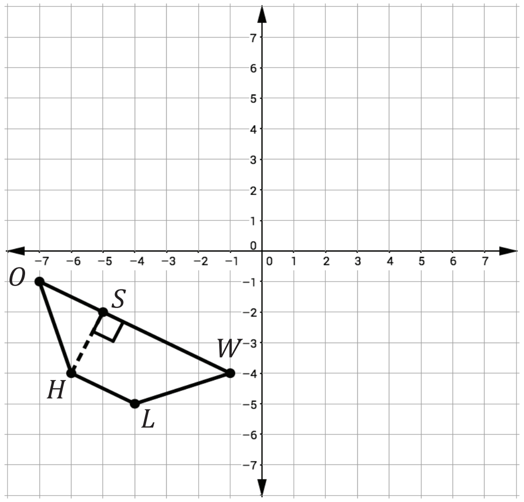


Given: ΔABC, and $\overline{BP}$ is parallel to $\overline{AC}$.
Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180°$

Statements	Reasons
1. ABC is a triangle.	**1.**
2. $\overline{BP} \parallel \overline{AC}$	**2.**
3. $m\angle 1 + m\angle 5 = m\angle PBA$	**3.**
4. $m\angle PBA + m\angle 4 = 180°$	**4.**
5. $m\angle 1 + m\angle 5 + m\angle 4 = 180°$	**5.**
6. $\angle 2 \cong \angle 4$; $\angle 3 \cong \angle 5$	**6.**
7. $m\angle 2 = m\angle 4$; $m\angle 3 = m\angle 5$	**7.**
8. $m\angle 1 + m\angle 2 + m\angle 3 = 180°$	**8.**

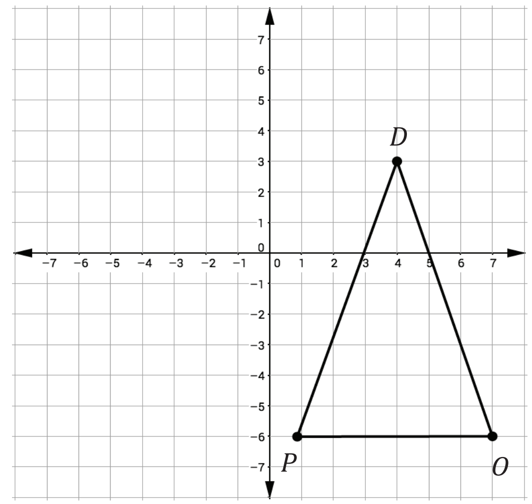
Section 5 – Topic 3
Triangles in the Coordinate Plane

1. Find the area of the trapezoid $HOWL$ plotted below. Round your answer to the nearest hundredth.



2. Triangle SBA has coordinates $S(15,-8)$, $B(-2,21)$, and $A(0,0)$. If the height of the triangle for the corresponding base $\overline{SB}$ is 8.89 units, then determine the perimeter and area of $\triangle SBA$. Round your answer to the nearest unit.

3. Consider $\triangle OPD$ in the coordinate system below.

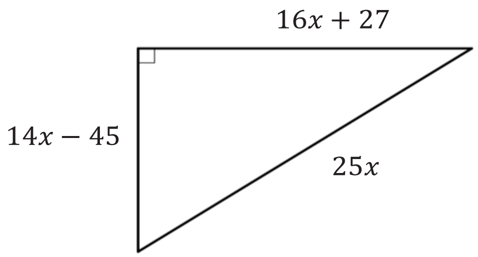


Part A: Find the approximate perimeter of the isosceles triangle $\triangle OPD$. Round your answer to the nearest hundredth.

Part B: If each block is equal to $25ft^2$, then determine the area of $\triangle OPD$.

4. Consider the right triangle below.

If the perimeter is $1{,}013$ units, find the value of x and the area of the triangle.

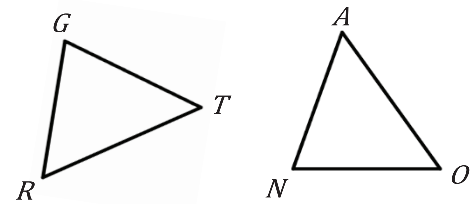


The value of x is ______________.

The area is ________________ square units.

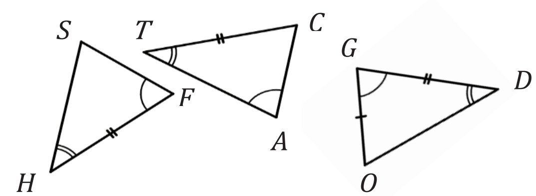
Section 5 – Topic 4
Triangle Congruence – SSS and SAS – Part 1

1. If $\Delta GTR \cong \Delta NAO$, then finish the following congruence statements and mark the corresponding congruent sides and the corresponding congruent angles.

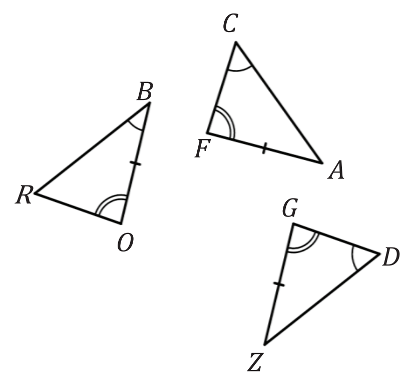


$\angle A \cong$ ______	$\overline{NO} \cong$ ______
______ $\cong \angle R$	______ $\cong \overline{GT}$
$\angle G \cong$ ______	$\overline{RT} \cong$ ______

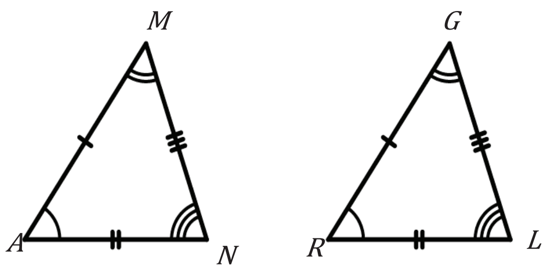
2. Name two triangles that are congruent by ASA.



3. Name two triangles that are congruent by AAS.



4. Complete the congruence statements for the triangles below.



$\Delta GRL \cong \Delta$______

$\angle N \cong \angle$______

$\overline{GL} \cong$ ______

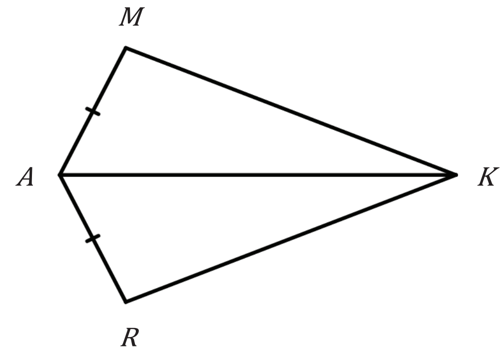
$\angle R + \angle N \cong \angle$______ $+ \angle$________

5. Circle the words in the highlighted fields that complete the sentence.

Part A: If two angles | sides and the included angle of one triangle are similar | congruent to two sides and the included angle of a second triangle, then the two triangles are congruent by the SSS | SAS | AAS | ASA congruence postulate.

Part B: If at least two | three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent by the SSS | SAS | AAS | ASA congruence postulate.

6. Consider the figure of the kite below.



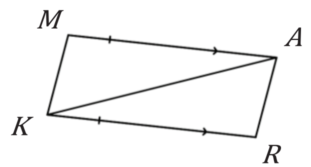
Part A: What information is needed to prove that the triangles above are congruent using the SSS Congruence Postulate?

Part B: What information is needed to prove that the triangles above are congruent using the SAS Congruence Postulate?

Section 5 – Topic 5
Triangle Congruence – SSS and SAS – Part 2

1. Ernie draws ΔMAR and ΔNIL where $\overline{MR} \cong \overline{NL}$, $\overline{MA} \cong \overline{NI}$, and $\angle A \cong \angle I$. Draw a sketch of ΔMAR and ΔNIL to determine if Ernie can use either SSS or SAS to prove the two triangles congruence. If the answer is no, explain what additional information the Ernie needs.

2. Consider quadrilateral $MARK$.

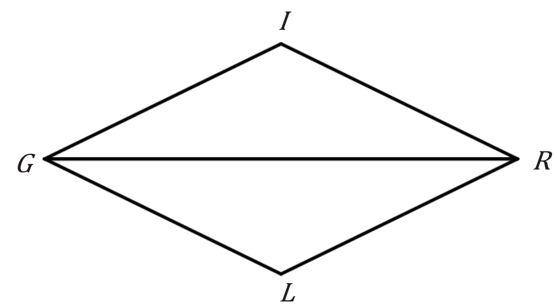


Given: $\overline{MA} \cong \overline{RK}$ and $\overline{MA} \parallel \overline{RK}$
Prove: $\Delta MAK \cong \Delta RKA$

Statements	Reasons
1.	**1.**
2.	**2.**
3.	**3.**
4.	**4.**
5.	**5.**

3. Rose claims that since $\Delta MTW \cong \Delta RFS$ are both equiangular triangles, then they must be congruent by the SSS Congruence Postulate. Determine whether Rose is correct or incorrect? Justify your answer.

4. Complete the paragraph proof.



Given: $\overline{GI} \cong \overline{IR} \cong \overline{GL} \cong \overline{RL}$
Prove: $\Delta GIR \cong \Delta GLR$

$\overline{GR}$ is congruent to $\overline{GR}$ by the ____________________ property of congruence. Since it is given that ____, then it is possible to say $\Delta GIR \cong \Delta GLR$ by ____________________ Congruence Postulate.

5. Draw ΔTHS and complete the sentences below.

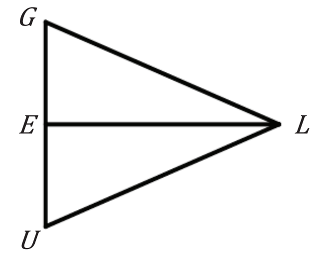
Part A: The angle that is included between $\overline{HT}$ and $\overline{ST}$ is _______________.

Part B: ____________ and _____________ include $\angle S$.

Section 5 – Topic 6
Triangle Congruence – ASA and AAS – Part 1

1. Complete the paragraph proof.

   Given: $\overline{LE}$ bisects $\angle L$ and $\overline{LE} \perp \overline{GU}$
   Prove: $\Delta LEG \cong \Delta LEU$



   $\overline{LE}$ bisects $\angle L$ is given. $\angle GLE$ is congruent to $\angle ULE$ by the definition of an ______________ ______________. $\overline{LE}$ is congruent to $\overline{LE}$ by the ________________ property of congruence. $\overline{LE}$ is perpendicular to $\overline{GU}$ is given, so $\angle GEL$ and $\angle UEL$ are right angles by the ____. Therefore, $\angle GEL$ is congruent to $\angle LEU$ because ______________________ are congruent.

   So, $\Delta LEG \cong \Delta LEU$ by __________________.

2. For the AAS Theorem to apply, which side of the triangle must be known?

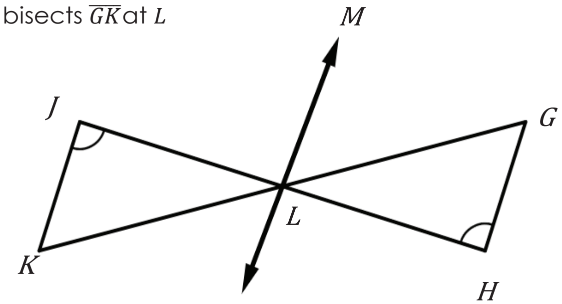
   Ⓐ the included side
   Ⓑ the longest side
   Ⓒ the shortest side
   Ⓓ a non-included side

3. For the ASA Postulate to apply, which side of the triangle must be known?

   Ⓐ the included side
   Ⓑ the longest side
   Ⓒ the shortest side
   Ⓓ a non-included side

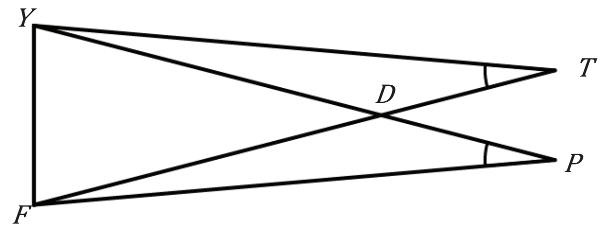
4. Complete the two-column proof below by writing in the statements and reasons.

Given: $\angle J \cong \angle H$ and line M bisects $\overline{GK}$ at L
Prove: $\Delta JLK \cong \Delta HLG$



Statements	Reasons
1.	**1.**
2.	**2.**
3.	**3.**
4.	**4.**
5.	**5.**

5. Consider the figure of overlapping triangles below.

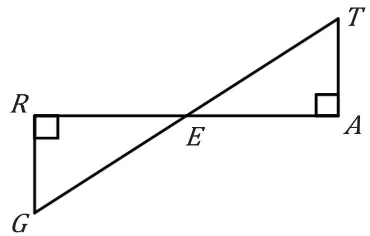


If it is given that $\angle T \cong \angle P$ and $\overline{YD} \cong \overline{FD}$, then what is needed to prove that $\Delta YDT \cong \Delta FDP$ using AAS?

Section 5 – Topic 7
Triangle Congruence – ASA and AAS – Part 2

1. Complete the two-column proof by writing in statements for the given reasons.

Given: $\angle R$ and $\angle A$ are right angles.
$\overline{RA}$ bisects $\overline{GT}$.
Prove: $\triangle GER \cong \triangle TEA$



Statements	Reasons
1.	**1.** Given
2.	**2.** All right angles are congruent
3.	**3.** Vertical angles are congruent
4.	**4.** Given
5.	**5.** Definition of bisector
6.	**6.** AAS congruence theorem

2. Consider triangle ΔNYM.

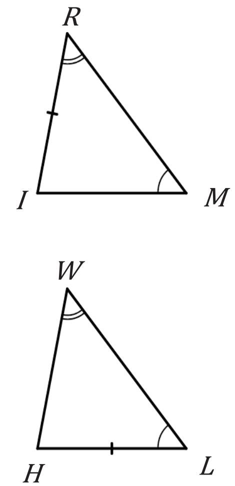
Part A: Which side is included between $\angle N$ and $\angle M$?

Part B: $\overline{YM}$ is between which angles?

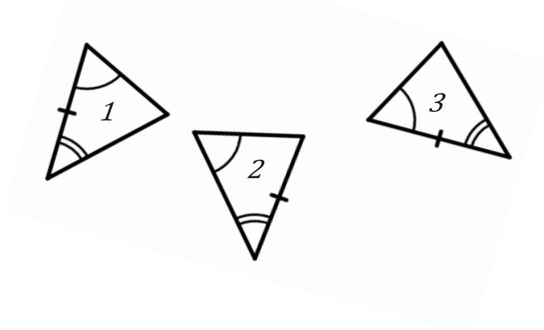
3. Consider the statement, $\Delta RIM \cong \Delta WHL$ for the figure on the right.

Which of the following statements represents and supports the statement above?

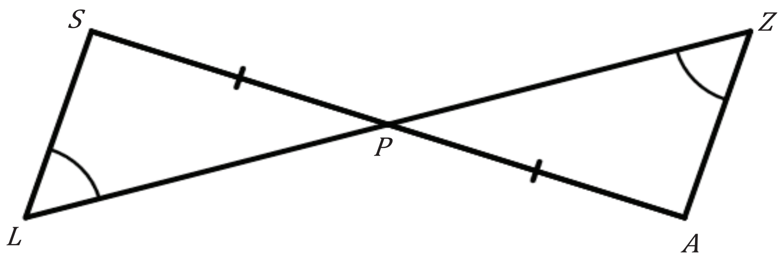
Ⓐ Yes, the triangles are congruent by ASA.
Ⓑ No, $\overline{RI}$ and $\overline{HL}$ are not corresponding sides.
Ⓒ Yes, the triangles are congruent by AAS.
Ⓓ No, $\angle B$ and $\angle U$ are not corresponding angles.



4. Determine which two triangles are congruent by ASA. Justify your answer.

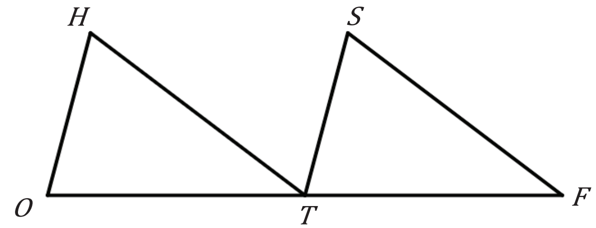


5. Determine if $\Delta SLP \cong \Delta ZAP$ are congruent. Justify your answer.



6. Complete the reasons for the statements below on the two-column proof below. (Hint: make markings on the triangles.)

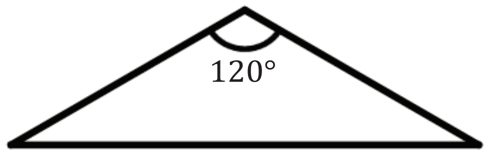
Given: $\overline{HT} \parallel \overline{SF}$, $\angle H \cong \angle S$ and $\overline{HT} \cong \overline{SF}$.
Prove: $\Delta HOT \cong \Delta STF$



Statements	Reasons
1. $\overline{HT} \parallel \overline{SF}$, $\angle H \cong \angle S$ and $\overline{HT} \cong \overline{SF}$	**1.**
2. $\angle F \cong \angle HTO$	**2.**
3. $\Delta HOT \cong \Delta STF$	**3.**

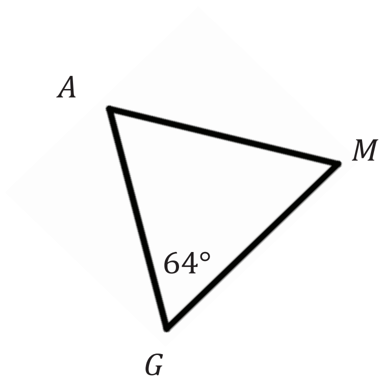
Section 5 – Topic 8
Base Angle of Isosceles Triangles

1. Consider the triangle below.



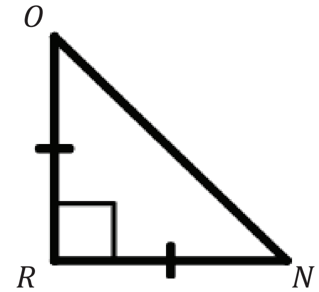
Determine the angle measure for the bottom two angles in order for the triangle to be classified as an isosceles triangle.

2. Consider the triangle below.

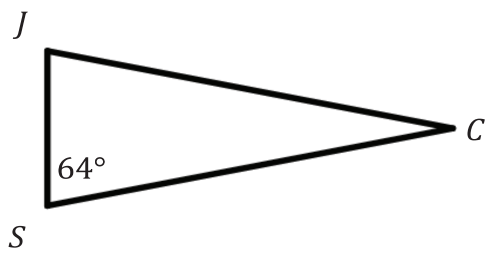


Determine the angle measure for $\angle A$ and $\angle M$ in order for ΔGAM to be classified as an isosceles triangle.

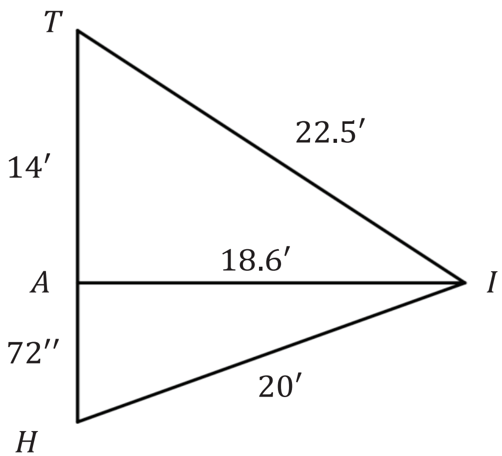
3. Determine the measures of $\angle N$ and $\angle O$ in ΔNOR below. List the degree measures from smallest to largest.



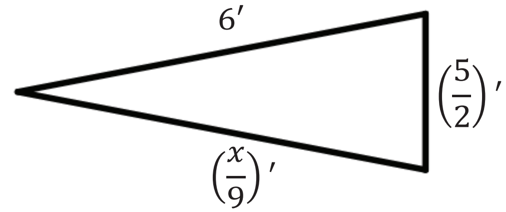
4. Determine the angle measure of the missing angles of ΔJCS below if ΔJCS is an isosceles triangle.



5. Identify the isosceles triangle below along with the base angles.



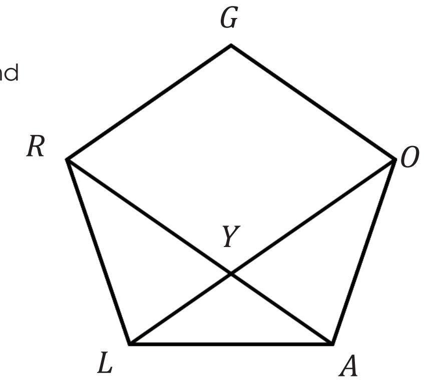
6. Determine the value of x for the following isosceles triangle.



7. Complete the two – column proof to the right with the word bank provided.

Given: Figure GOALR is equilateral and equiangular.

Prove: ΔYLA is an isosceles triangle.



A. Equilateral figures have congruent sides

B. Isosceles Triangle Definition

C. Reflexive Property

D. Side – Angle – Side

E. Corresponding Parts of Corresponding Triangles are Congruent

F. An equiangular figure has all angles that are congruent

G. Transitive Property

H. Base Angle Theorem

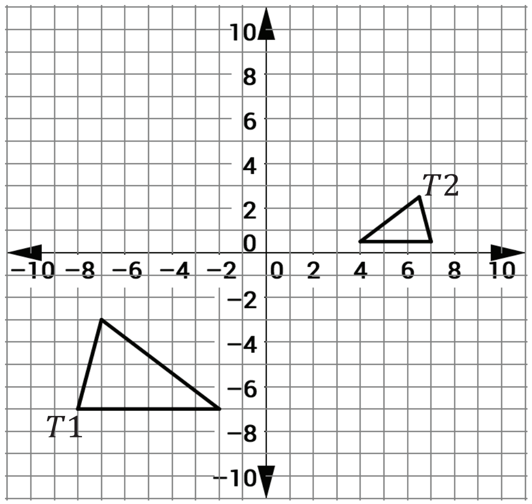
I. Angle – Angle – Side

J. Angle Addition Postulate

Statement	Reasons
1. GOALR is equilateral and equiangular.	1. Given
2. $\overline{RL} \cong \overline{OA}$	2.
3. $\overline{LA} \cong \overline{LA}$	3.
4. $\angle RLA \cong \angle OAL$	4.
5. $\Delta RLA \cong \Delta OAL$	5.
6. $\angle YLA \cong \angle YAL$	6.
7. $\overline{YL} \cong \overline{YA}$	7.
8. ΔYLA	8.

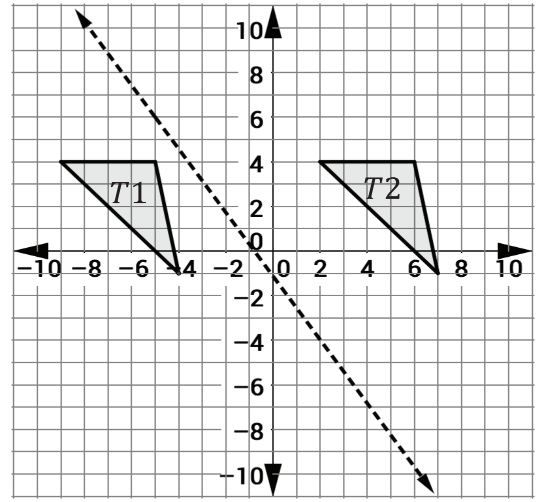
Section 5 – Topic 9
Using the Definition of Triangle Congruence in Terms of Rigid Motions

1. Consider the diagram below.



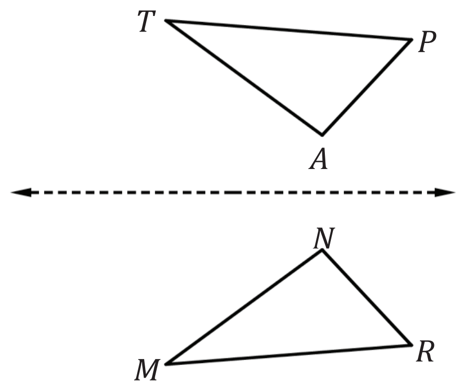
Find a combination of rigid motions that will map $\Delta T1$ onto $\Delta T2$ and determine if $\Delta T1 \cong \Delta T2$.

2. Consider the diagram below.



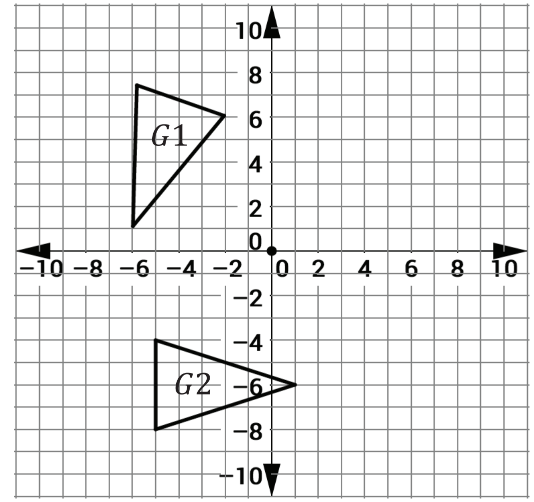
Determine the rigid motion that will map $\Delta T1$ onto $\Delta T2$.

3. Consider the diagram below.



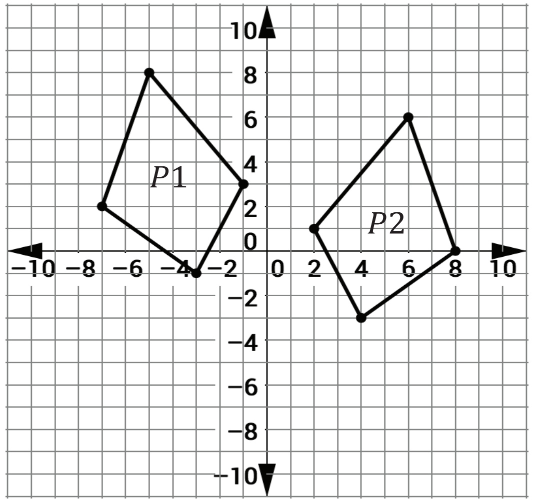
Your best friend determined that $\Delta MNR \cong \Delta APT$; however, you overheard someone say that $\Delta RNM \cong \Delta PAT$. Determine who is correct and explain why the other answer is incorrect.

4. Consider the diagram below.



Determine how $\Delta G1$ can be mapped onto $\Delta G2$ using the point at the origin.

5. Mr. Burton gave his students an extension problem. Consider the diagram below.

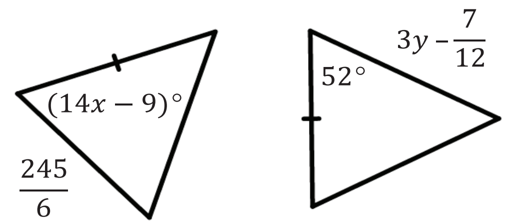


Part A: Determine if there is a rigid motion combination that transforms $\Delta P1$ onto $\Delta P2$. If it does, then describe the rigid motion.

Part B: Make a conclusion about rigid motions and every polygon congruence.

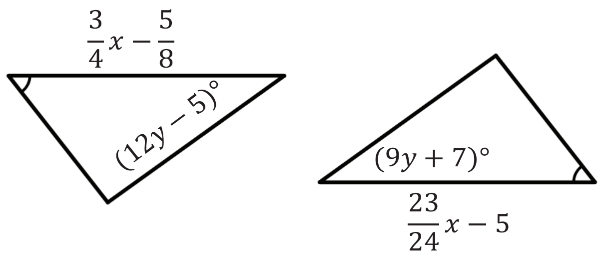
Section 5 – Topic 10
Using Triangle Congruency to Find Missing Variables

1. Consider the figures below.



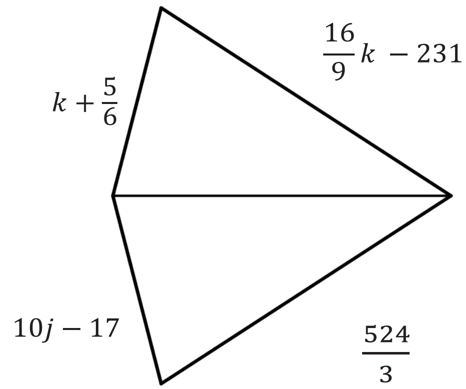
Find the value of x and y in order to prove that the two triangles are congruent by the SAS Congruence Postulate. Justify your work.

2. Consider the figures below.



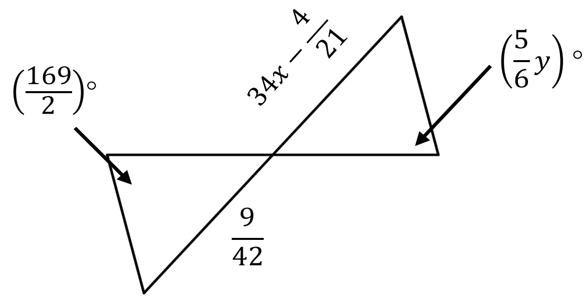
Find the value of x and y in order to prove that the two triangles are congruent using the ASA Congruence Postulate. Justify your work.

3. Consider the figure below.



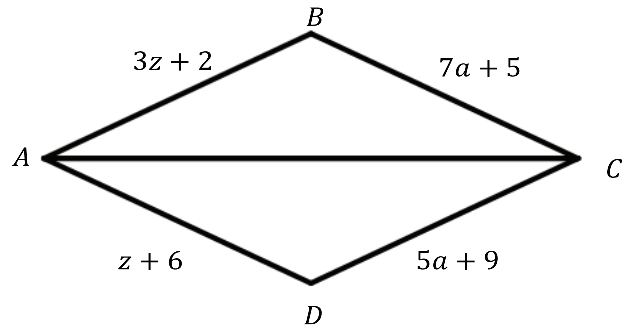
Find the values of k and j that prove the two triangles are congruent using the SSS Congruence Postulate.

4. Consider the figure below.



Find the values of x and y that prove the two triangles are congruent using the AAS Congruence Theorem. Justify your work.

5. Consider the figure below.



Part A: If $\overline{AB} \cong \overline{AD}$ and $\overline{BC} \cong \overline{DC}$ then because $\overline{AC} \cong \overline{AC}$ by the [Ⓐ transitive / Ⓑ symmetric / Ⓒ supplement / Ⓓ reflexive] property of congruence, it is possible to determine that $\Delta ABC \cong \Delta ADC$ by [Ⓐ AAS. / Ⓑ ASA. / Ⓒ SAS. / Ⓓ SSS.]

Part B: What are the values of z and a?

Section 6: Triangles - Part 2
Student Learning Plan

Topic Number	Topic Name	Date Completed	Study Expert(s)	Check Your Understanding Score
1	Triangle Similarity – Part 1			
2	Triangle Similarity – Part 2			
3	Triangle Midsegment Theorem – Part 1			
4	Triangle Midsegment Theorem – Part 2			
5	Triangle Inequalities			
6	More Triangle Proofs			
7	Inscribed and Circumscribed Circles of Triangles			
8	Medians in a Triangle			

What did you learn in this section? What questions do you still have?

Who was your favorite Study Expert for this section? Why?

Section 6 – Topic 1
Triangle Similarity – Part 1

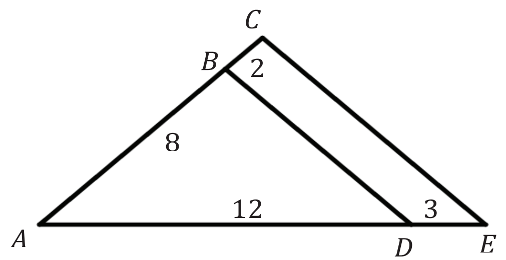
1. Consider the statement below:

   Congruent triangles are always similar.

   Which of the following statements is an example of the statement above? Select all that apply.

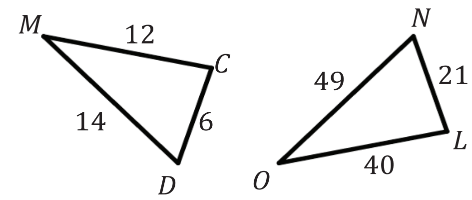
   - ☐ Angles are the same, but sides are proportional to each other.
   - ☐ Sides are the same size.
   - ☐ A dilation of a scale factor $\neq 1$.
   - ☐ Corresponding angles and corresponding sides are congruent.
   - ☐ A dilation of a scale factor of 1.

2. Determine if the two triangles are similar. If so, write a similarity statement for the triangles. Justify your answer.

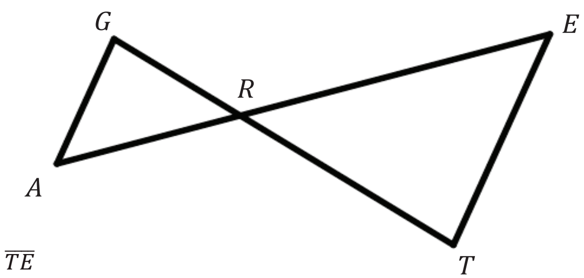


Statement	Reason
1. $\angle A \cong \angle A$	**1.**
2. $\frac{AB}{AC} =$ ______	**2.**
3. $\frac{AD}{AE} =$ ______	**3.**
4. $\Delta ABD \sim \Delta ACE$	**4.**

3. Are the following triangles similar? Justify your answer.



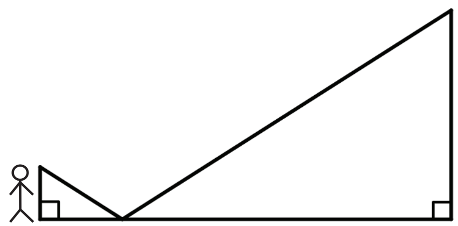
4. Consider the following figure and proof.



Given: $\overline{GA} \parallel \overline{TE}$
Prove: $\Delta GRA \sim \Delta TRE$

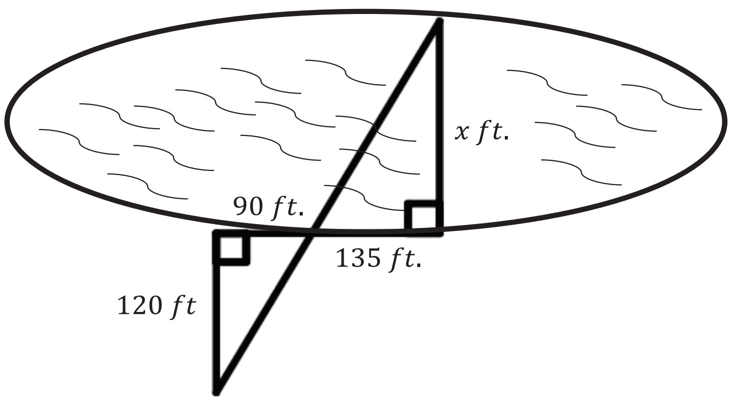
Statement	Reason
1.	**1.** Given
2.	**2.** Alternate Interior Angles are Congruent
3.	**3.** Alternate Interior Angles are Congruent
4. $\Delta GRA \sim \Delta TRE$	**4.**

5. Before rock climbing, Fernando, who is 5.5 feet tall, wants to know how high he will climb. He places a mirror on the ground and walks six feet backwards until he can see the top of the cliff in the mirror.

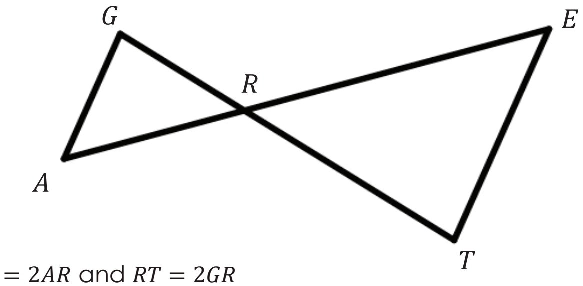


Determine the similarity theorem or postulate that you can use to determine the height of the cliff.

6. Determine what similarity postulate or theorem we can use to determine the value of x, the width of the river.



7. Consider the following figure and proof.

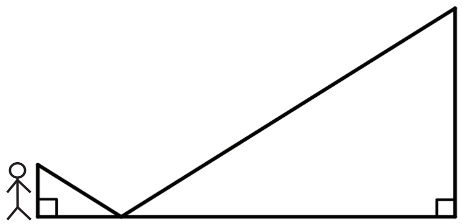


Given: $RE = 2AR$ and $RT = 2GR$
Prove: $\Delta GAR \sim \Delta TER$

Statement	Reason
1.	**1**. Given
2.	**2**. Vertical Angles
3.	**3**. Proportionality of Sides
4. $\Delta GAR \sim \Delta TER$	**4.**

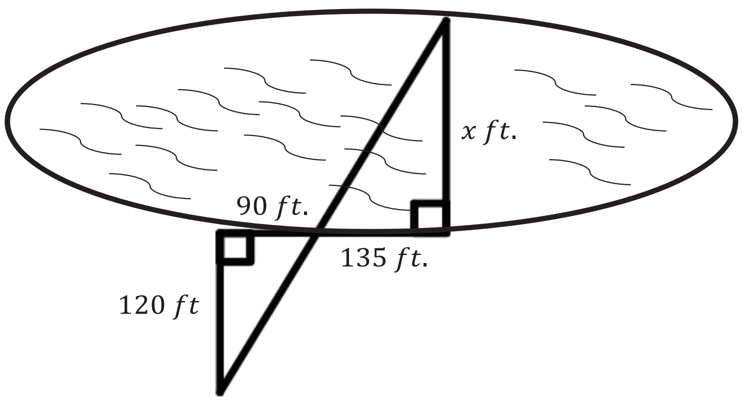
Section 6 – Topic 2
Triangle Similarity – Part 2

1. Before rock climbing, Fernando, who is $5.5\ ft.$ tall, wants to know how high he will climb. He places a mirror on the ground and walks six feet backwards until he can see the top of the cliff in the mirror.

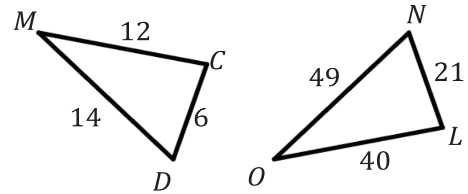


If the mirror is 34 feet from the cliff side, determine the height of the cliff.

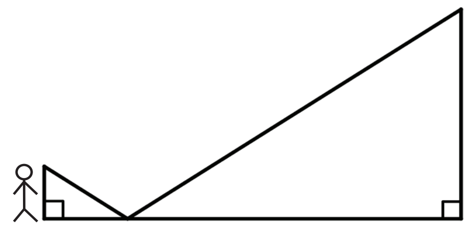
2. Determine the width of the pond.



3. The following triangles are not similar. Determine the ratio between ΔMCD and ΔOLN. How could you change the measurement(s) to make them similar?



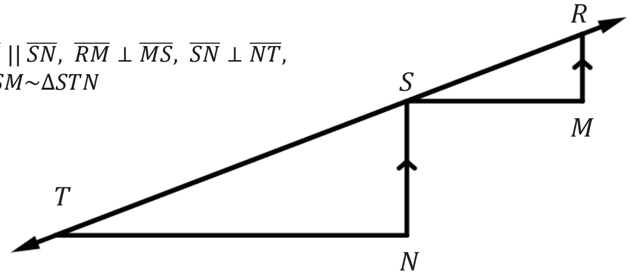
4. Basketball star Mumford (a six-foot senior forward) places a mirror on the ground $x\ ft.$ from the base of a basketball goal. He walks backward four feet until he can see the top of the goal, which he knows is 10 feet tall. Determine the how far the mirror is from the basketball goal. Justify your answer.



5. A $1.4\ m$ tall child is standing next to a flagpole. The child's shadow is $1.2\ m$ long. At the same time, the shadow of the flagpole is $7.5\ m$ long. How tall is the flagpole?

6. Consider the following figure and proof.

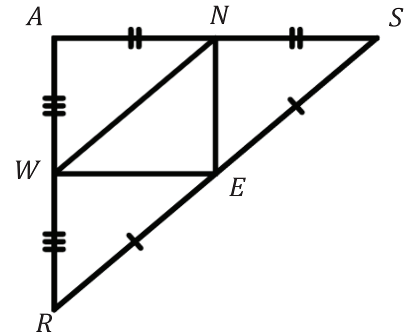
Given: $\overline{RM} \parallel \overline{SN}$, $\overline{RM} \perp \overline{MS}$, $\overline{SN} \perp \overline{NT}$,
Prove: $\Delta RSM \sim \Delta STN$



Statement	Reason
1.	**1.**
2.	**2.**
3.	**3.**
4.	**4.**
5.	**5.**

Section 6 – Topic 3
Triangle Midsegment Theorem – Part 1

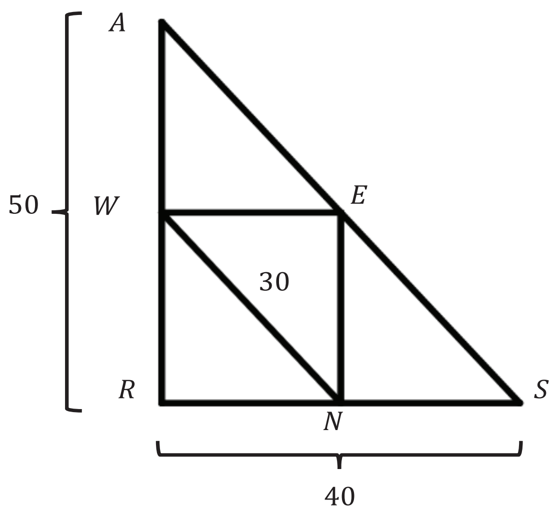
1. Consider ΔASR.



Write the three pairs of parallel segments in ΔASR.

$\overline{WE}$	_______||_______	$\overline{RS}$
$\overline{NE}$	_______||_______	$\overline{AS}$
$\overline{WN}$	_______||_______	$\overline{RA}$

2. In ΔASR, W, E, and N are midpoints. Determine the lengths of each segment using the number bank.



50	30	20
40	25	60

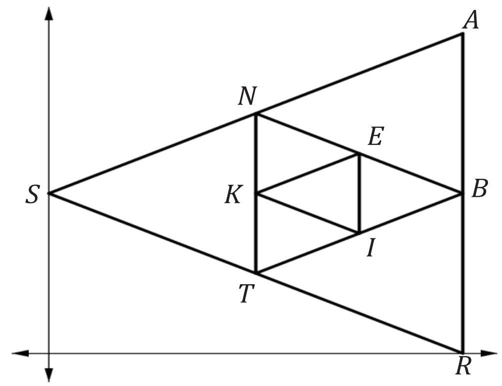
Part A: $AS =$ _______

Part B: $WE =$ _______

Part C: $EN =$ _______

Part D: $AR + RS - AS =$ _______

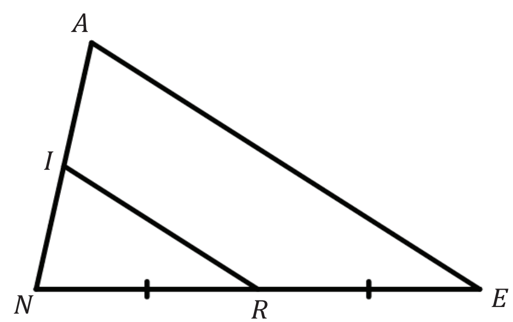
3. In the diagram below, R is located at $(24, 0)$, N is located at $(12, 18)$, T is located at $(12, 6)$, and E is located at $(18, 15)$. Assume that N, K, T, E, I, and B are midpoints.



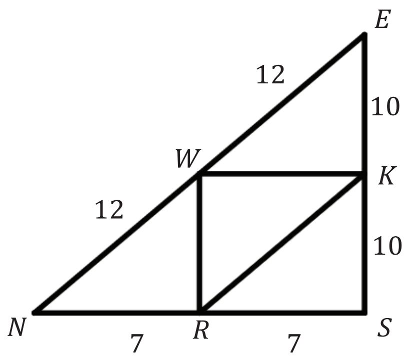
Complete the following table.

Vertices	S	K	I	B	A
Coordinates					

4. As an answer to a test, Monica sees the figure below and determines that $\overline{IR} \| \overline{AE}$. Monica's teacher marked it as incorrect. Determine the error in Monica's reasoning.

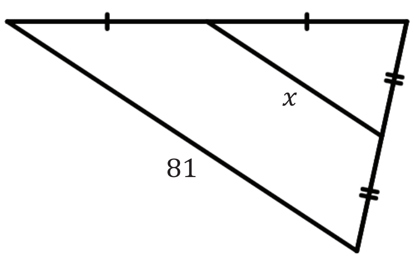


5. Identify three pairs of parallel segments in each diagram and write the pairs in the blanks.

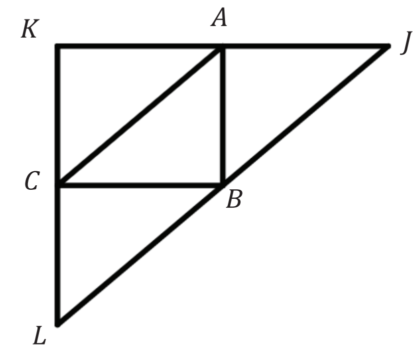


$\overline{WK}$	_______||_______	$\overline{SN}$
$\overline{WR}$	_______||_______	$\overline{NE}$
$\overline{RK}$	_______||_______	$\overline{ES}$

6. Determine the value of x.



7. Consider the following figure and proof.

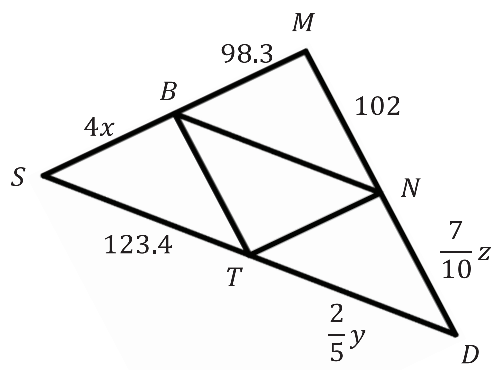


Given: A bisects $\overline{JK}$, C bisects $\overline{KL}$, B bisects $\overline{JL}$,
Prove: $\Delta JKL \sim \Delta CBA$

Statement	**Reason**
1.	**1.**
2.	**2.**
3.	**3.**
4.	**4.**
5.	**5.**
6.	**6.**

Section 6 – Topic 4
Triangle Midsegment Theorem – Part 2

1. In ΔSMD, point B is the midpoint of $\overline{SM}$, point N is the midpoint of $\overline{MD}$, and point T is the midpoint of $\overline{DS}$.

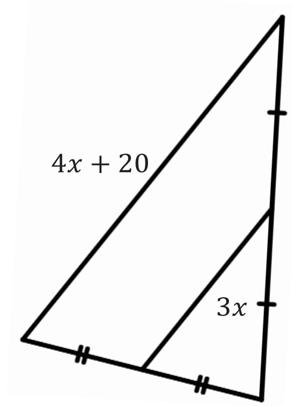


Part A: Find the length of SM.

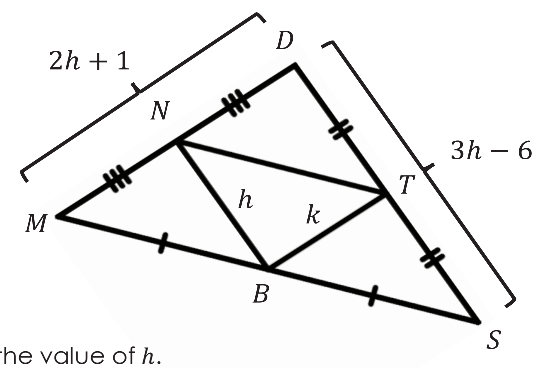
Part B: Determine the value of $x+y+z$.

Part C: Find the value of $NT+TB-BN$.

2. Determine the value of x.



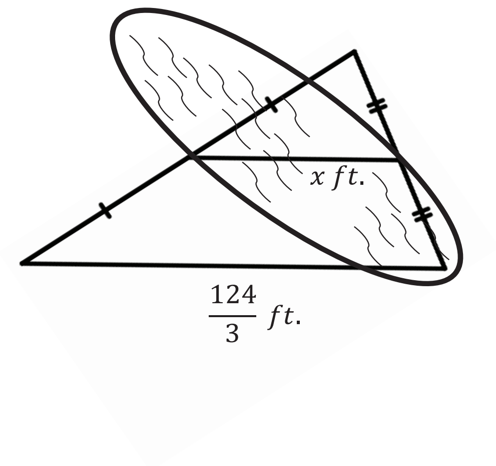
3. Consider the following figure.



Part A: Determine the value of h.

Part B: Determine the value of k.

4. Determine the length across the river, x, to the nearest hundredth.



5. The coordinates of the vertices of a triangle are $M(-4,1)$, $A\ (3,3)$, and $N\ (2,-3)$.

Part A: Determine the coordinate of J, the midpoint of $\overline{MA}$.

Part B: Determine the coordinate of L, the midpoint of $\overline{AN}$.

Part C: Prove that $JL = \frac{1}{2}MN$.

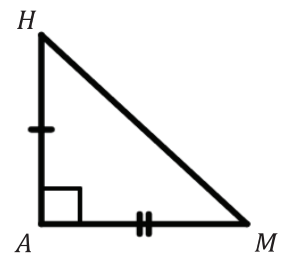
Section 6 – Topic 5
Triangle Inequalities

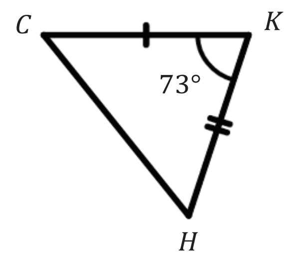
1. Consider the following triangle side lengths and determine if the triangle could exist. Justify your answer.

Part A: $21, 18, 17$

Part B: $3, 12, 8$

2. Consider the following figure.

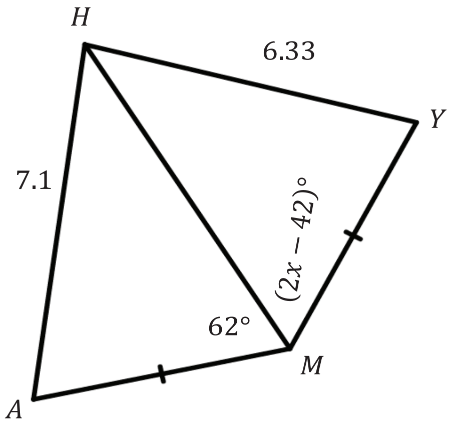




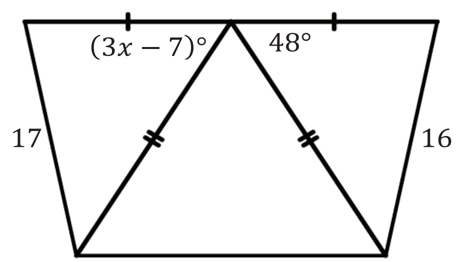
Determine which of the following statements must be true.

Ⓐ $HA < CH$
Ⓑ $HM > CH$
Ⓒ $AM = CK$
Ⓓ $MH < HC$

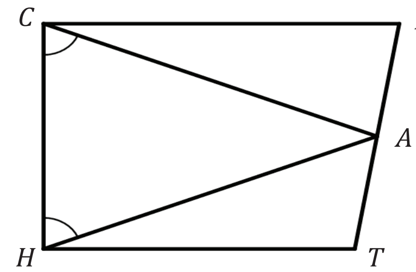
3. Determine the range of possible values of x.



4. Find the range of possible values of x.



5. Consider the following figure and proof.



Given: A is the midpoint of $\overline{ET}$,
$m\angle CHA = m\angle HCA$, $m\angle EAC > m\angle TAH$

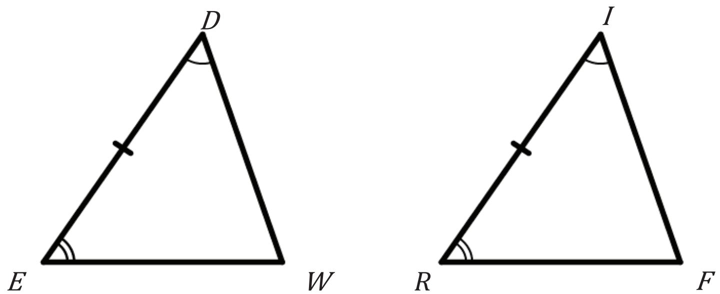
Prove: $CE > HT$

Complete the two-column proof below

Statement	Reason
1. $m\angle CHA = m\angle HCA$	**1**. Given
2. $CA = HA$	**2**.
3. A is the midpoint of $\overline{ET}$	**3**. Given
4. $\overline{EA} \cong \overline{AT}$	**4**.
5.	**5**. Congruent segments have equal length.
6. $m\angle EAC > m\angle TAH$	**6**. Given
7.	**7**.

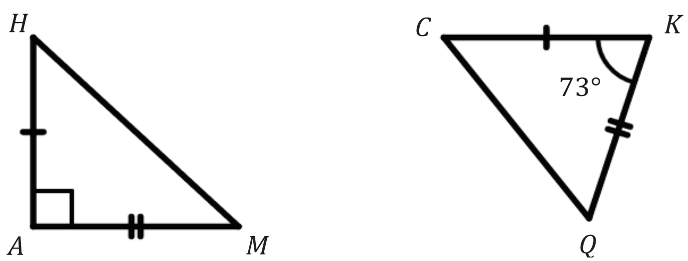
Section 6 – Topic 6
More Triangle Proofs

1. Consider the figures below.



Abony used the above diagram to conclude that $\overline{DW} \cong \overline{IF}$. Explain the rationale behind Abony's conclusion and justify whether or not her conclusion is correct.

2. Consider the following figure.

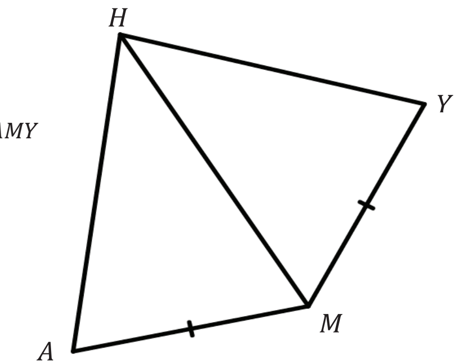


Jazeel used the above diagram to conclude that $\overline{HM} \cong \overline{CQ}$. Explain the rationale behind Jazeel's conclusion and justify whether or not his conclusion is correct.

3. Consider the following figure.

Given: $\overline{AM} \cong \overline{YM}$; $\overline{HM}$ bisects $\angle AMY$

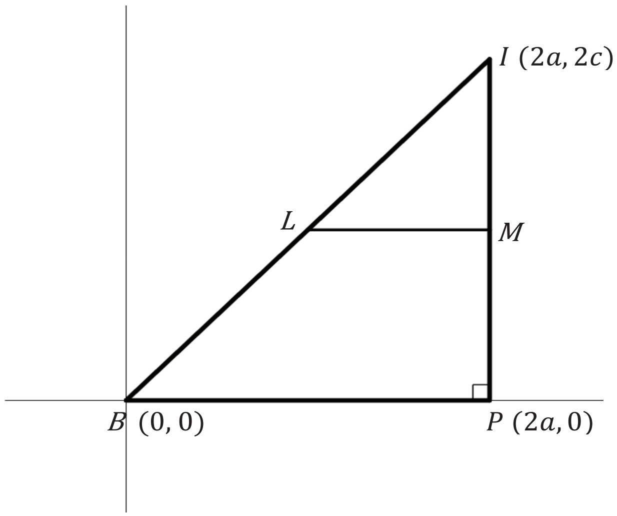
Prove: $\angle A \cong \angle Y$



Based on the above figure and the information below, complete the following two-column proof.

Statements	Reasons
1. $\overline{AM} \cong \overline{YM}$	**1.**
2. $\overline{HM}$ bisects $\angle AMY$	**2.**
3. $\angle AMH \cong \angle YMH$	**3.**
4. $\overline{HM} \cong \overline{HM}$	**4.**
5. $\Delta AHM \cong \Delta YHM$	**5.**
6. $\angle A \cong \angle Y$	**6.**

4. Consider the figure below.



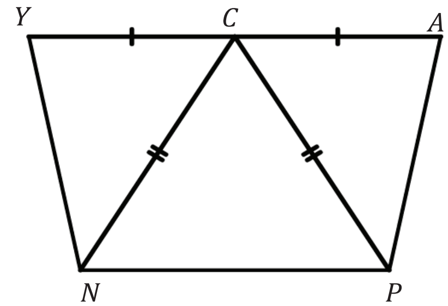
The above figure shows ΔBIP where L is the midpoint of $\overline{BI}$ and M is the midpoint of $\overline{IP}$.

Part A: Prove that $\triangle BIP \sim \triangle LIM$.

Part B: Prove algebraically that the area of $\triangle LIM$ is one-fourth the area of $\triangle BIP$.

Part C: Justify whether or not CPCTC can be used in a scenario like the one presented in the above figure.

5. Consider the figure below.

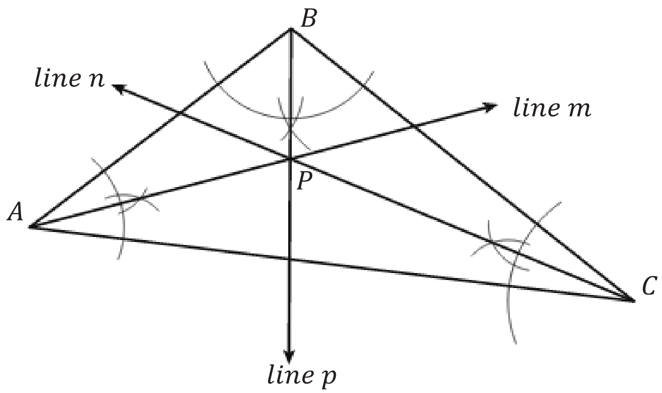


Part A: Based on the above figure and a given statement, Cassidy was able to conclude that $\triangle NYC \cong \triangle PAC$ because of SAS. Determine the given statement.

Part B: Prove that $\overline{NY} \cong \overline{PA}$ both theoretically and by applying transformations.

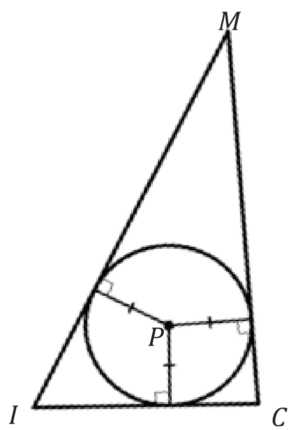
Section 6 – Topic 7
Inscribed and Circumscribed Circles of Triangles

1. Consider the figure below.



Dante argues that point P is the circumcenter of the triangle. Determine if Dante is correct and justify your answer.

2. Consider the following figure and prove that the three angle bisectors of the internal angles of $\triangle CMI$ are concurrent in point P.

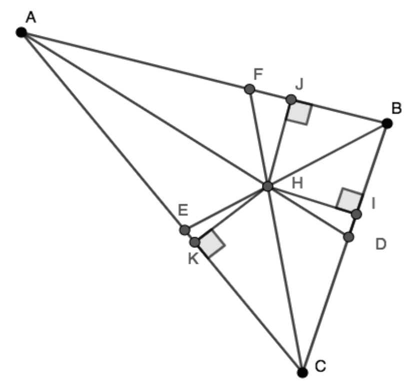


3. Ms. Calcutta, the owner of a business park, is adding a recycling station for every three office buildings. To make it easier for the tenants, she is going to use the circumcenter of the three office buildings to determine the placement of the recycling stations. By doing this, it will be easier to get to the station, because it will be equidistant from the three office buildings.

Part A: Justify the rationale behind Ms. Calcutta's decision to use the circumcenter.

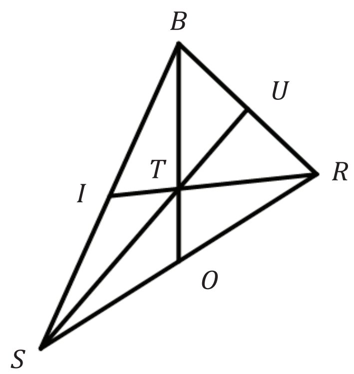
Part B: There is a garbage dumpster located exactly at the midpoint of each pair of office buildings. Suppose there are sidewalks connecting each office building to each other and to each recycling station. What is the relationship between the building-to-building sidewalk and the station-to-dumpster sidewalk?

4. Consider the figure below and mark the line segments and angles that are congruent, given that H is the incenter of the triangle.



Section 6 – Topic 8
Medians in a Triangle

1. Consider the figure below.

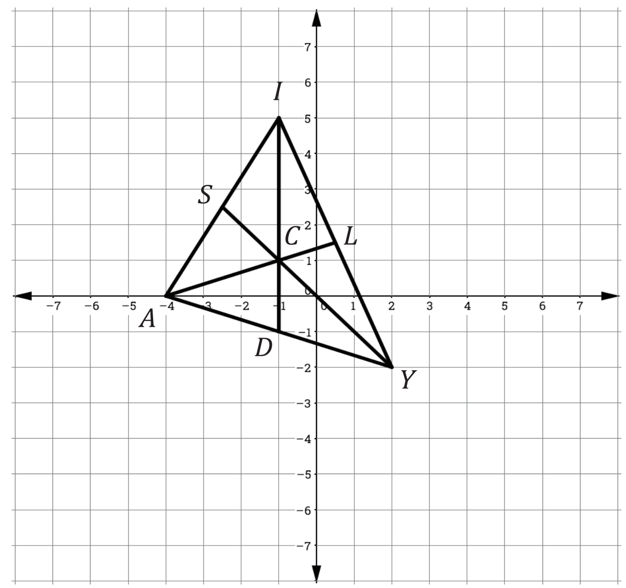


$\overline{IR}, \overline{US}$, and $\overline{OB}$ are all medians of $\triangle BRS$, and T is the centroid. $IR = 10.8'$, $BT = 4.5'$, $UT = 3.15'$. Find RT, TI, OB, and US.

2. Describe the similarities and differences between the circumcenter of a triangle and the centroid of a triangle.

3. Describe the similarities and differences between the incenter of a triangle and the centroid of a triangle.

4. Consider the triangle below.



Prove that c is the centroid of $\triangle AIY$.

Section 7: Right Triangles and Trigonometry
Student Learning Plan

Topic Number	Topic Name	Date Completed	Study Expert(s)	Check Your Understanding Score
1	The Pythagorean Theorem			
2	The Converse of the Pythagorean Theorem			
3	Proving Right Triangles Congruent			
4	Special Right Triangles: $45° - 45° - 90°$			
5	Special Right Triangles: $30° - 60° - 90°$			
6	Right Triangles Similarity – Part 1			
7	Right Triangles Similarity – Part 2			
8	Introduction to Trigonometry – Part 1			
9	Introduction to Trigonometry – Part 2			
10	Angles of Elevation and Depression			
11	Segments in Regular Polygons			
12	Area of Other Polygons			
Honors 1	Proving and Applying the Law of Sines			
Honors 2	Proving and Applying the Law of Cosines			

**Honors resources are available online.*

What did you learn in this section? What questions do you still have?

Who was your favorite Study Expert for this section? Why?

Section 7 – Topic 1
The Pythagorean Theorem

1. Tony wants ivy to grow on his white picket fence in a certain direction. He decides to run a metal wire diagonally from the ground level at one end of the fence to the top of the other side of the fence row. The section of the fence in the front of his house is 10 feet long and has a height of 27 inches.

   Determine the length of the wire.

2. Alice leaves her house and walks to school. She walks 45 meters south and 336 meters east. How far is Alice from her house?

3. The walls of square storage room in a warehouse are 300 feet long. What is the distance from one corner to the other corner of the storage room?

4. Alejandro has three ladders that are 15, 10, and 12 feet in length. If he is trying to reach a window that is 8 feet from the ground, then…

   Part A: How far from the wall does the bottom of the ladder need to be if Alejandro wants to use the 15-foot ladder?

   Part B: How far from the wall does the bottom of the ladder need to be if Alejandro wants to use the 10-foot ladder?

   Part C: How far from the wall does the bottom of the ladder need to be if Alejandro wants to use the 12-foot ladder?

5. If the hypotenuse of a right triangle is 125 units long and the short leg adjacent to the right angle is 32 units long, determine the length of the long leg of the triangle.

Section 7 – Topic 2
The Converse of the Pythagorean Theorem

1. Prove that each example below is a Pythagorean triple.

   Part A: $77, 420, 427$

   Part B: $279, 440, 521$

   Part C: $39, 760, 761$

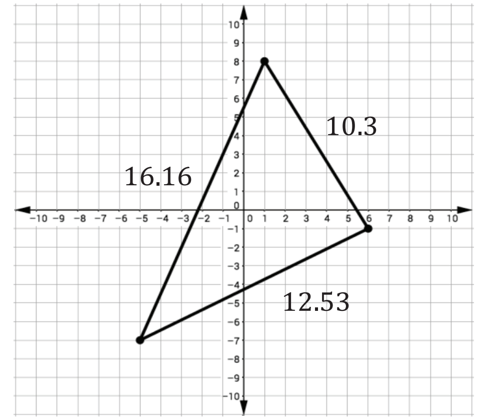
2. Determine if the triples below are right triangles.

   Part A: $290, 696, 750$

   Part B: $514, 684, 855$

   Part C: $450, 1080, 1170$

3. Consider the following figure.



   Determine if the triangle on the coordinate plane is a right triangle by using the converse of the Pythagorean theorem.

4. If a triangle has side lengths of $56, 90$, and 106, then determine which number correlates to the legs and hypotenuse.

   56 is a _______________. 90 is a _____________. 106 is a ________________.

5. How can you determine which of the three numbers is the hypotenuse? Once the hypotenuse is identified, then does it matter which length you substitute for the a and b in $a^2 + b^2 = c^2$? Justify your answer.

6. For any triangle with side lengths a, b, and c, if $a^2 + b^2 = c^2$, then the triangle is a right triangle; if $a^2 + b^2 > c^2$, then the triangle is an acute triangle; and if $a^2 + b^2 < c^2$, then the triangle is an obtuse triangle.

   Determine if the following lengths of a given triangle is an acute, right, or obtuse triangle.

   Part A: $30, 40, 50$ is a(n) _______________ triangle.

   Part B: $0.3, 0.4, 0.6$ is a(n) __________________ triangle.

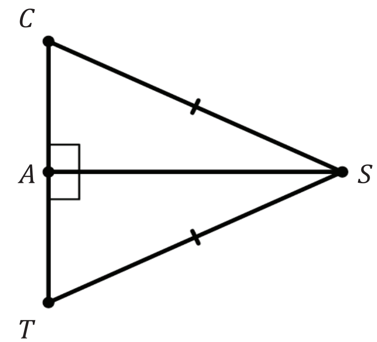
   Part C: $11, 12, 15$ is a(n) ____________________ triangle.

7. Determine which of the following triples is not a right triangle. Select all that apply.

   ☐ $90, 215, 243$
   ☐ $60, 144, 156$
   ☐ $40, 75, 85$
   ☐ $20, 22, 29$
   ☐ $33, 56, 65$

Section 7 – Topic 3
Proving Right Triangles Congruent

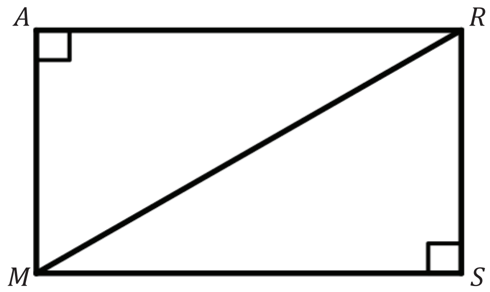
1. Use the diagram to prove that $\Delta CAS \cong TAS$.



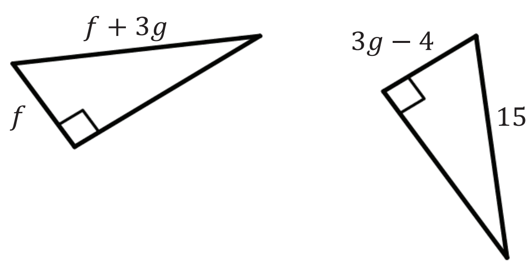
Given: $\overline{CS} \cong \overline{ST}$ and $m\angle CAS = m\angle TAS = 90°$
Prove: $\triangle CAS \cong \triangle TAS$

Statement	Reason
1.	**1.**
2.	**2.**
3.	**3.**
4.	**4.**
5.	**5.**
6.	**6.**

2. Determine the congruence information that is needed to show that the two triangles are congruent by the HL Theorem in two different ways.

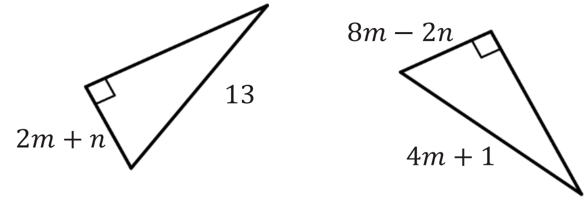


3. Consider the following diagram.



Find the values of f and g that prove the two triangles are congruent by the HL Theorem.

4. Consider the following diagram.



Find the values of m and n that prove the two triangles are congruent by the HL Theorem.

Section 7 – Topic 4
Special Right Triangles: $45° - 45° - 90°$

1. At the local baseball diamond, the distance from home base to second base is 100 feet.

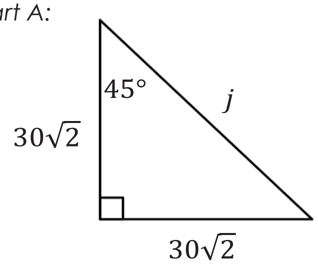
   Part A: Determine the distance from home base to first base.

   Part B: If Sammy hits a homerun, what is the distance that she has to run in order to run around all the bases?

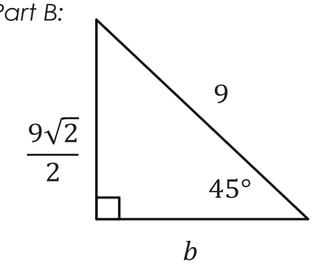
2. A decent-sized square plot of land in town is one acre (1 acre $= 43560$ sq.ft.). If Mr. Pearson wants to play football with his son Connor, then how far can they throw the football from corner to corner?

3. Use the Pythagorean Theorem or knowledge on special right triangles to find the missing variable in the following triangles.

   Part A:

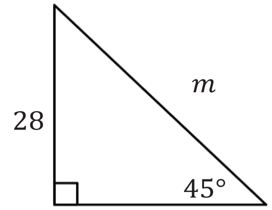


   Part B:

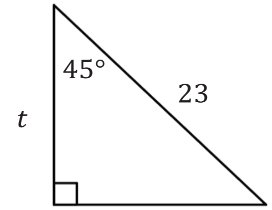


4. Use the Pythagorean Theorem or knowledge on special right triangles to find the missing variable in the following triangles.

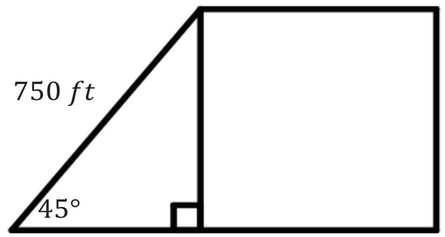
   Part A:



   Part B:



5. Consider the image below of a pasture at McDonald's Farm.

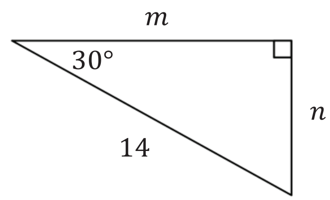


   Part A: If McDonald wants to fence in the square pasture and barbed-wire fencing costs $\$1.15$ per foot, then determine the cost to fence in the pasture.

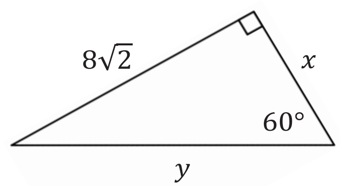
   Part B: If McDonald wants to cover the square pasture in fertilizer, then he needs to determine the area of the pasture. What is the area of the pasture to the nearest hundredth?

Section 7 – Topic 5
Special Right Triangles: $30° - 60° - 90°$

1. Determine the value of the missing sides for the following triangle.

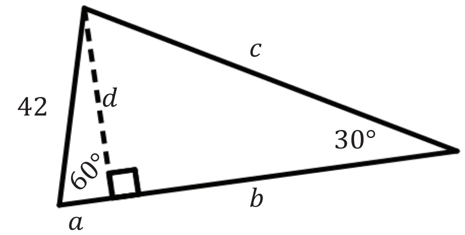


2. Determine the value of the missing sides for the following triangle.



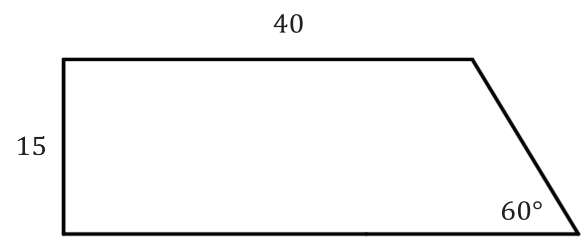
3. The length of the side opposite the $30°$ angle of a $30-60-90$ is $323\ ft$. Determine the lengths of the other two sides.

4. Determine the value of each variable in the figure below. Keep answers in simplest radical form.



5. An equilateral triangle has a height of $52\ cm$. Determine the length of each side to the nearest hundredth of a centimeter.

6. Consider the following figure.

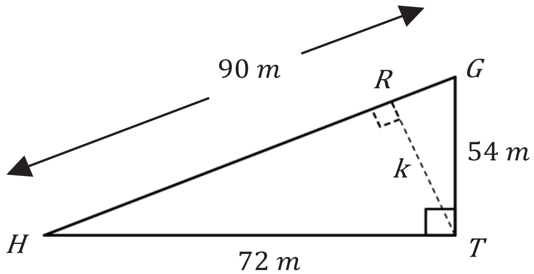


Part A: Determine the perimeter of the figure above.

Part B: Determine the area of the figure above.

Section 7 – Topic 6
Right Triangles Similarity – Part 1

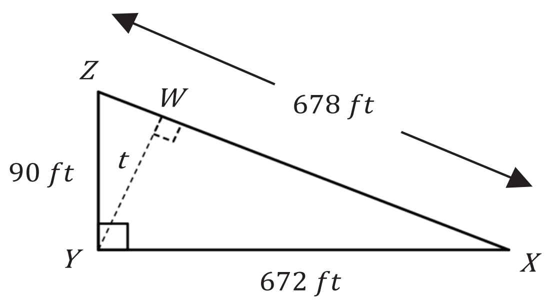
1. Consider the following diagram.



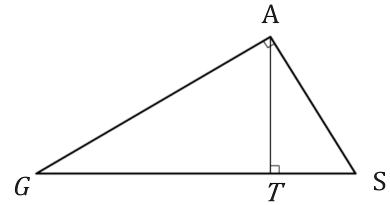
Part A: Identify the similar triangles in the above diagram.

Part B: Find the value of k in the above diagram.

2. Determine the value of the variable in the figure below.

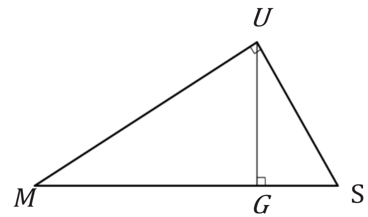


3. Complete the following blanks to complete the criteria for the Geometric Mean Theorem: Altitude Rule.



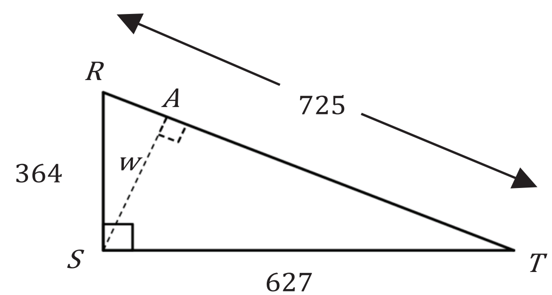
$$\frac{\overline{GT}}{\square} = \frac{\square}{\overline{TS}}$$

4. Complete the following blanks to complete the criteria for the Geometric Mean Theorem: Leg Rule.



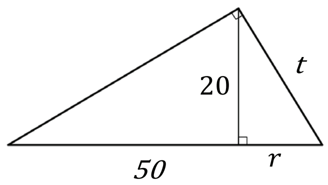
$$\frac{\overline{MS}}{\square} = \frac{\square}{\overline{GS}}$$ or $$\frac{\overline{MS}}{\square} = \frac{\square}{\overline{MG}}$$

5. Determine the value of the variable in the figure below.

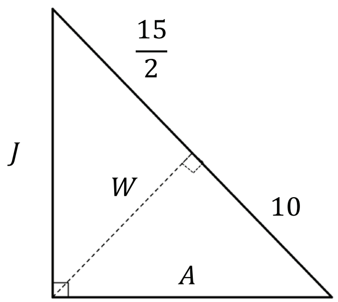


Section 7 – Topic 7
Right Triangles Similarity – Part 2

1. Determine the variables in the figure below.

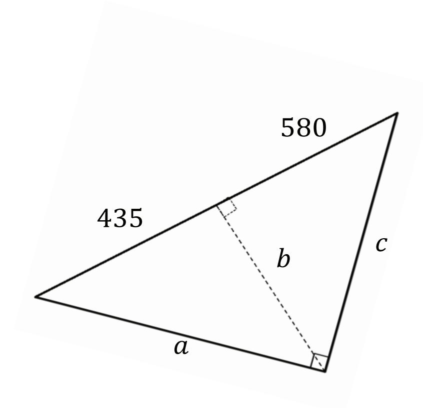


2. Consider the figure below.



Find the values of J, W, and A.

3. Consider the diagram below.



Determine the values of a, b, and c.

4. A school, a hospital, and a supermarket are located at the vertices of a right triangle formed by three highways. The school and hospital are 14.7 miles apart. The distance between the school and the supermarket is 8.82 miles, and the distance between the hospital and the supermarket is 11.76 miles.

   A service road will be constructed from the main entrance of the supermarket to the highway that connects the school and hospital. What is the shortest possible length for the service road? Round your answer to the nearest tenth.

5. A huge forest has a fence in the shape of a right triangle with sides measuring $850\sqrt{3}\ miles, 850\ miles$, and $1700\ miles$. During the winter season, the forest rangers open up another road to bypass the snow that could accumulate. The road will connect the right angle of the forest to the hypotenuse.

   Determine the shortest possible length for the bypass road.

Section 7 – Topic 8
Introduction to Trigonometry – Part 1

1. Match the correct trigonometric value to the correct ratio.

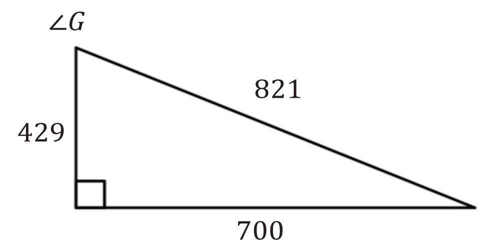
cot	tan	csc
sec	sin	cos

$$\_\_\_\_\_\_ = \frac{leg\ opposite\ to\ the\ angle}{hypotenuse}$$

$$\_\_\_\_\_\_ = \frac{leg\ adjacent\ to\ the\ angle}{hypotenuse}$$

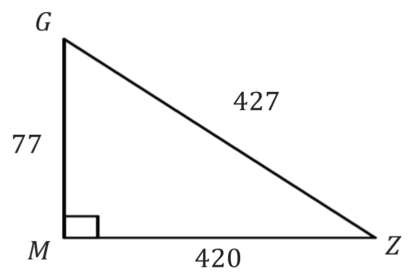
$$\_\_\_\_\_\_ = \frac{leg\ opposite\ to\ the\ angle}{leg\ adjacent\ to\ the\ angle}$$

2. Consider the figure below and determine the value of sine, cosine, and tangent of $\angle G$.



$\sin(G) =$________ $\cos(G) =$________ $\tan(G) =$________

3. Consider the figure below.



Part A: Determine $\sin(G)$.

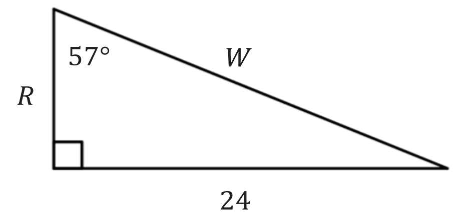
Part B: Determine $\tan(Z)$.

Part C: Determine $\cos(Z)$.

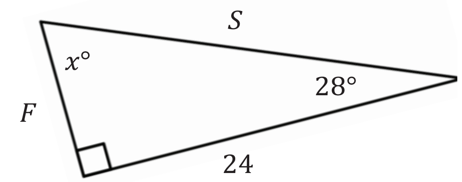
Part D: Determine $\sin(M)$.

Part E: Make a conjecture about the sine value of **any** right angle.

4. Determine the unknown values, R and W, in the figure below.



5. Determine all the unknown values of the figure below.



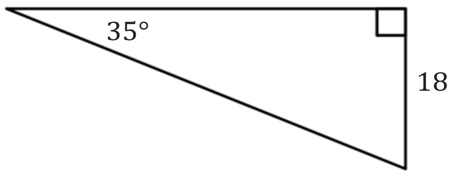
6. Match the sides to the correct trigonometry function. Each word can be used more than once.

opposite adjacent hypotenuse

$\cos =$ ______________ $\tan =$ ______________ $\sin =$ ______________

Section 7 – Topic 9
Introduction to Trigonometry – Part 2

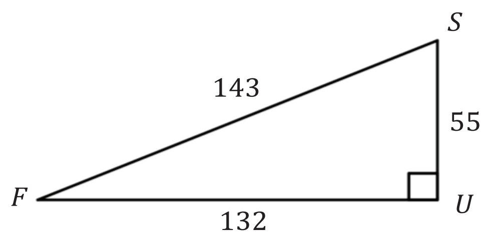
1. Consider the triangle below.



Part A: Determine the perimeter of the triangle to the nearest hundredth.

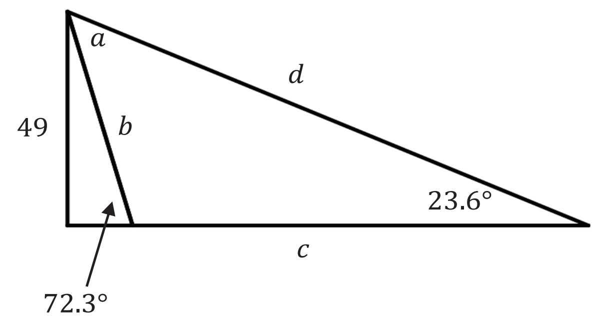
Part B: Determine the area of the triangle to the nearest hundredth.

2. Consider the triangle ΔFSU below.

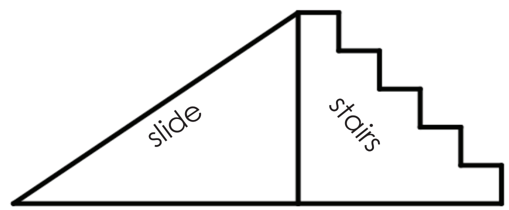


Using what you know about trigonometric functions, determine the angle measure of $\angle S$ and $\angle F$.

3. Consider the right triangle in the figure below and determine the value of $a, b, c,$ and d.



4. At the museum, there is a *Jack in the Beanstalk* play area with stairs and a slide representing Jack's journey up to the giant's lair. Consider the diagram of the play area below.



Each step is two feet long and three feet high. If Jack is standing 4 feet from the base of the stairs, determine the angle of elevation from the ground where Jack is standing to the top of the stairs. Round your answer to the nearest tenth of a degree.

5. A ladder is leaning against a wall and makes an $82°$ angle of elevation with the ground.

   Part A: If the base of the ladder is 4 feet from the wall, then how long is the ladder? Round to the nearest tenth.

   Part B: Determine how high up the wall the ladder reaches.

6. A right triangle has a hypotenuse of length 59 and one leg of length 23.

   Part A: Determine the angle measure of the other two acute angles in the triangle. Round to the nearest hundredth.

   Part B: Determine the side length of the other leg.

Section 7 – Topic 10
Angles of Elevation and Depression

1. Mary's kite is flying above the beach at the end of a 75 –meter string. If the angle of elevation to the kite measures $68°$ and Mary is holding the kite 1.2 meters off the ground, then determine the height of Mary's kite in flight.

2. A weather balloon is launched from the ground at a location that is 250 yards from a 65 foot royal palm tree. What is the minimal angle of elevation at which the balloon must take off in order to avoid hitting the tree? Assume that the balloon flies in a straight line and the angle of elevation stays constant.

3. Juliet stands at the window of her apartment so that her eyes are 38.4 feet above the ground. Juliet sees her boyfriend down the street a few blocks away. She knows, based on the number of city blocks, that her boyfriend is at a distance of 156.45 feet away from the building. Determine the angle of depression of Juliet's sight to her boyfriend on the ground.

4. A straight road to the top of a mountain is $25{,}525$ feet long and makes a $44°$ angle of elevation from sea level. Determine the height of the mountain.

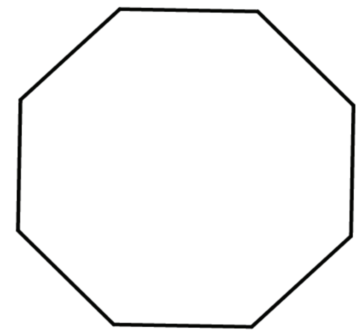
5. Airplanes use Distance Measuring Equipment (DME) to measure the distance from the plane to a nearest radar station. If the distance from a plane to a radar station is 160 miles and the angle of depression is $34°$, find the number of ground miles from a point directly below the plane to the radar station.

6. A traffic camera sits on top of a tower that has a height of $50\ ft$. The angles of depression of two cars on a straight road at the same level as that of the base of the tower and on the same side of the tower are $25°$ and $40°$. Calculate the the distance between the two cars.

7. A man on the deck of a ship is $14\ ft$. above water level. He observes that the angle of elevation of the top of a cliff is $40°$ and the angle of depression of the base is $20°$. Find the distance of the cliff from the ship and the height of the cliff if the base of the cliff is at sea level.

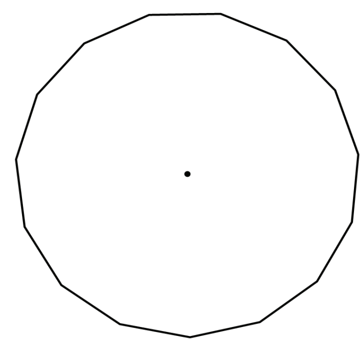
Section 7 – Topic 11
Segments in Regular Polygons

1. Consider the regular polygon.



How many diagonals does the regular polygon have? Justify your answer.

2. Consider the regular polygon.

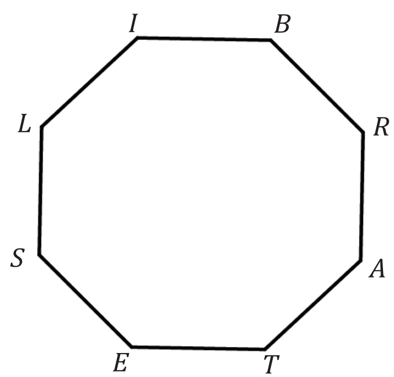


How many diagonals does the regular polygon have? Justify your answer.

3. Consider a regular $23-$gon. How many diagonals does the regular polygon have?

4. A regular polygon has 20 diagonals. Determine how many sides there are.

5. Consider the regular octagon $LIBRATES$.

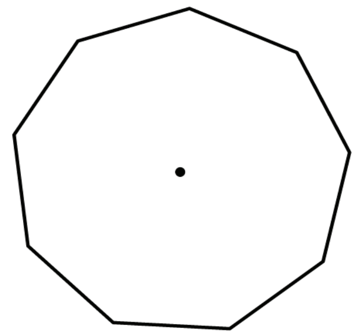


Part A: How many diagonals does the regular polygon have?

Part B: Diagonal $\overline{IE}$ and $\overline{IA}$ form quadrilateral $IATE$. What type of quadrilateral is $IATE$ and what are the measures of its interior angles?

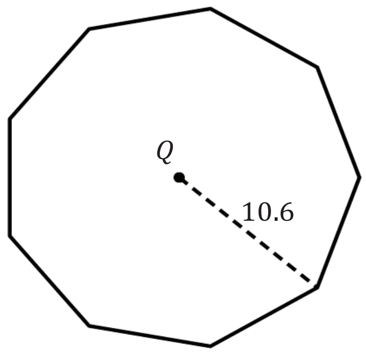
6. A regular polygon has 27 diagonals. Determine how many sides there are.

7. Consider the regular nonagon with a given center below.



If the nonagon above has a side length of m inches, then determine the length of the apothem, a.

8. Use the regular polygon with center Q to answer the questions below.



Part A: What is the measure of the apothem in the above polygon?

Part B: What is the measure of each side of the regular polygon?

9. A regular decagon has a radius of $14'$.

Part A: Determine the length of the apothem.

Part B: Determine the perimeter of the decagon.

10. A regular $16-$gon has a radius of 10 meters.

Part A: Determine the length of the apothem.

Part B: Determine the perimeter of the polygon.

11. *Vanderbilt Tile Creations* is designing a tile pattern for the lobby in the National Headquarters of Veterans of Foreign Wars (VFW) in Kansas City, MO. The design is a regular heptagon with a 30 foot diameter that has a statue of a kneeling soldier in the center.

Part A: Determine the distance from the kneeling soldier to one of the vertices of the heptagon.

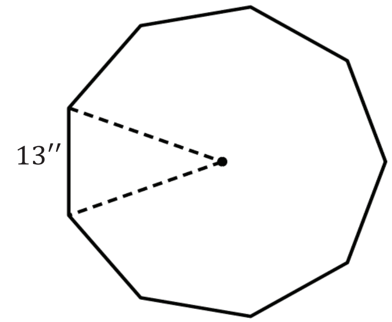
Part B: Determine the distance from the kneeling soldier to the middle of one of the sides of the heptagon.

Part C: Determine the perimeter of the heptagon.

12. The tomb of Gunbad-Kabud in Maragheh, Iran is the shape of a decagonal tower and has a side length of approximately 8 feet. Determine the distance from the center of the tomb to the outside wall.

Section 7 – Topic 12
Area of Other Polygons

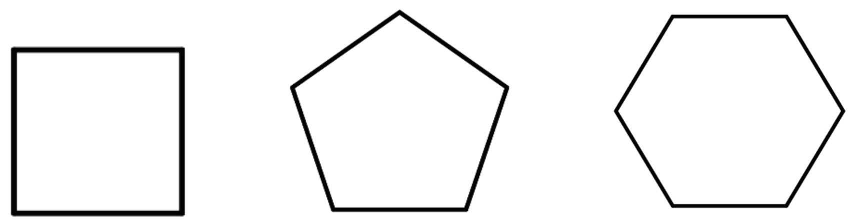
1. Use the regular polygon with a center to answer the questions.



Part A: Determine the area of the dotted triangle.

Part B: Determine the area of the entire regular polygon.

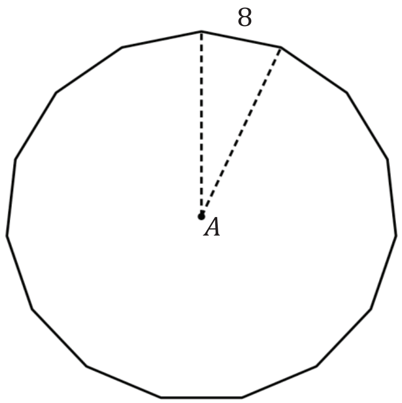
2. Consider the following polygons with side length five.



Part A: Prove the hexagon has the greatest area.

Part B: How many times larger is the pentagon's area than the square's area? How many times larger is the hexagon's area than the square's area?

3. Use the regular polygon with center, A, to answer the questions.



Part A: Determine the area of the dotted triangle.

Part B: Determine the area of the entire regular polygon.

4. Determine the area of a regular hendecagon that has a side length of seven inches and an apothem of 12.42 inches.

5. *DJI* and *Hubsan* are two drone companies that want to introduce two new drones. *DJI* wants to create a six-bladed drone with the battery pack located at the center. *DJI*'s drone has an apothem of 10 inches, covering an area of 423 square inches. *Hubsan*'s new drone is in the shape of an octagon with an apothem of 8 inches, covering an area of 410 square inches. Assume both drones are regular polygons.

   Determine which drone has the biggest perimeter.

6. *TileMath Inc*. is the new "hip" tile company in town. Bass Pro Shop headquarters in Springfield, MO asked *TileMath, Inc.* to create two options for a new tile design in the lobby. The first option is in the shape of a hendecagon with a side length of nine feet, and the materials cost \$3.00 per square foot. The second option is in the shape of a nonagon with a side length of 11 feet, and the materials cost \$4.50 per square foot.

   Determine which option for Bass Pro is more cost effective.

Section 8: Quadrilaterals
Student Learning Plan

Topic Number	Topic Name	Date Completed	Study Expert(s)	Check Your Understanding Score
1	Introduction to Quadrilaterals – Part 1			
2	Introduction to Quadrilaterals – Part 2			
3	Introduction to Quadrilaterals – Part 3			
4	Parallelograms – Part 1			
5	Parallelograms – Part 2			
6	Rectangles			
7	Rhombi			
8	Squares			
9	Kites			
10	Trapezoids			
11	Quadrilaterals in Coordinate Geometry – Part 1			
12	Quadrilaterals in Coordinate Geometry – Part 2			
Honors 1	Midsegment of Trapezoids			

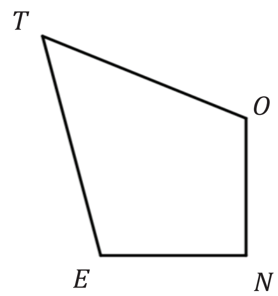
**Honors resources are available online.*

What did you learn in this section? What questions do you still have?

Who was your favorite Study Expert for this section? Why?

Section 8 – Topic 1
Introduction to Quadrilaterals – Part 1

1. Consider the quadrilateral $TONE$ below.



Part A: Name three sets of consecutive vertices.

Part B: Name three sets of consecutive sides.

Part C: Name two sets of opposite sides.

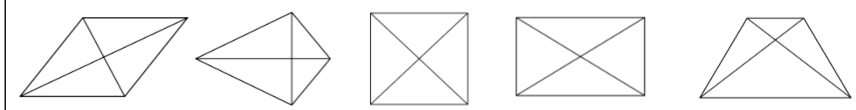
Part D: Name three sets of adjacent angles.

Part E: Name two sets of opposite angles.

Part F: Name two diagonals.

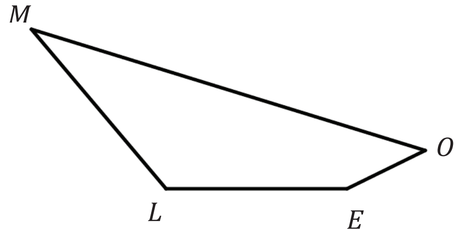
2. Match the characteristic from the word bank to the correct quadrilateral. Words from word bank will repeat.

Word Bank			
A. Congruent	**B.** Parallel	**C.** Not Parallel	**D.** Not Congruent
E. Different	**F.** Supplementary	**G.** Complementary	**H.** Perpendicular
I. One set is parallel.	**J.** One set is Congruent.	**K.** Base angles are congruent.	**L.** Bisect each other
M. 90°	**N.** Opposite Angles are congruent.	**O.** Adjacent Angles are supplementary.	**P.** Adjacent sides are perpendicular.
Q. **R.**	**S.**	**T.**	**U.**



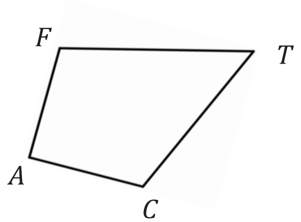
Characteristics of Quadrilaterals	**Kite**	**Rectangle**	**Rhombus**	**Parallelogram**	**Square**	**Isosceles Trapezoid**
Opposite Sides						
Angles						
Diagonals						
Adjacent Sides						
Shape						

3. Determine which of the following pairs is an adjacent side for quadrilateral $MOLE$ below.



Ⓐ $\overline{MO}$ and $\overline{LE}$
Ⓑ $\overline{EO}$ and $\overline{ME}$
Ⓒ $\overline{LE}$ and $\overline{OL}$
Ⓓ $\overline{ML}$ and $\overline{LE}$

4. Consider the quadrilateral $FACT$ below.



Part A: Determine which of the following pairs is an opposite angle.

Ⓐ $\angle F$ and $\angle T$
Ⓑ $\angle A$ and $\angle C$
Ⓒ $\angle C$ and $\angle T$
Ⓓ $\angle T$ and $\angle A$

Part B: Determine which of the following pairs is an adjacent angle.

Ⓐ $\angle F$ and $\angle C$
Ⓑ $\angle A$ and $\angle T$
Ⓒ $\angle C$ and $\angle T$
Ⓓ $\angle T$ and $\angle A$

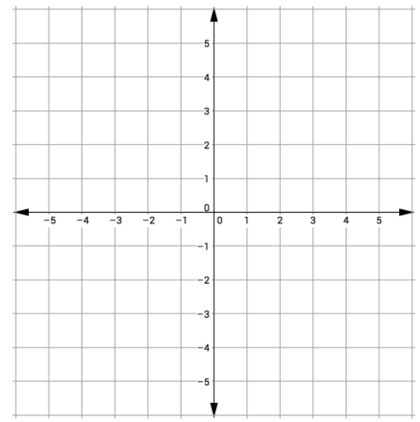
Section 8 – Topic 2
Introduction to Quadrilaterals – Part 2

1. The quadrilateral $CONR$ on a coordinate plane has the following characteristics.

   $\overline{CO}$ can be represented by the equation $y = 9 + 2x$ where $-4 \leq x \leq -3$.

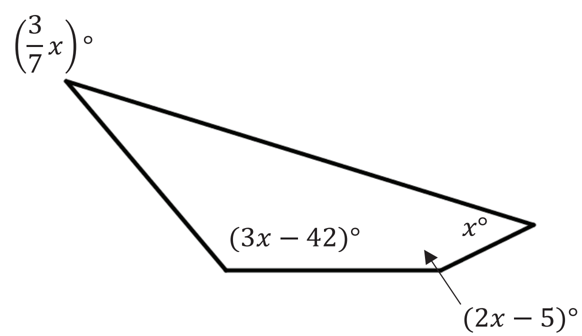
   $\overline{RN}$ can be represented by the equation $y - 2x = -1$ where $-1 \leq x \leq 2$.

   Part A: Graph the figure represented by the information given.



   Part B: Describe the type of quadrilateral represented above.

2. Find the measure of each interior angle. Round to the nearest hundredth.



3. Match the correct area formula to the polygon. You can use polygons more than once.

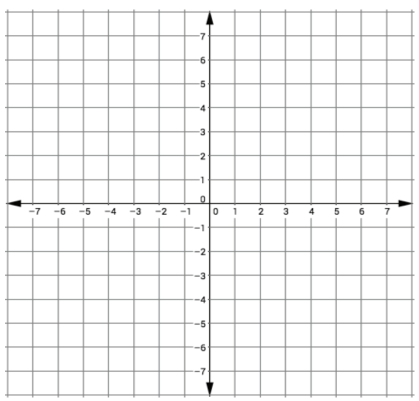
	Area	Polygon
________	$A = \left(\frac{b_1 + b_2}{2}\right) * h$	**A.** Kite
________	$A = lw$	**B.** Square
________	$A = bh$	**C.** Rhombus
________	$A = s^2$	**D.** Rectangle
________	$A = \frac{1}{2} d_1 d_2$	**E.** Parallelogram
		F. Isosceles Trapezoid

4. The quadrilateral $SMUG$ on a coordinate plane has the following characteristics.

$\overline{SM}$ can be represented by the equation $x + 4y = 7$ where $3 \le x \le 7$.

$\overline{UG}$ can be represented by the equation $y = \frac{1}{4}x - \frac{5}{2}$ where $2 \le x \le 6$.

Part A: Graph the figure represented by the information given above.



Part B: Describe the type of quadrilateral represented on the coordinate plane.

5. Classify the following descriptive statements as quadrilaterals or non-quadrilaterals. If non-quadrilaterals, explain why.

Part A: A figure with 4 different pairs of opposite angles.

O Quadrilateral O Non-quadrilateral

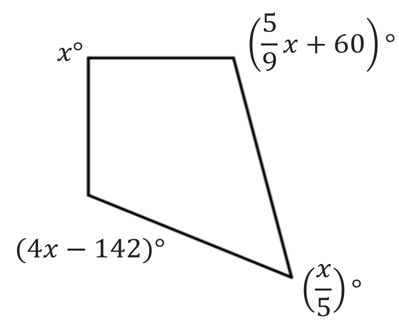
Part B: A figure with $m\angle H = 85.3, m\angle O = 123.8, m\angle T = 105.75$, and $m\angle L = 46.15$.

O Quadrilateral O Non-quadrilateral

Part C: A figure with $m\angle H = \frac{572}{16}, m\angle O = \frac{653}{5}, m\angle T = \frac{3023}{20}$, and $m\angle L = \frac{85}{2}$.

O Quadrilateral O Non-quadrilateral

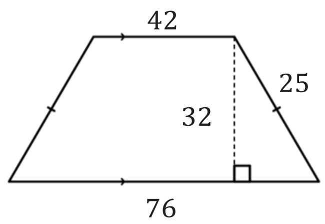
6. Find the measure of each interior angle. Round to the nearest hundredth.



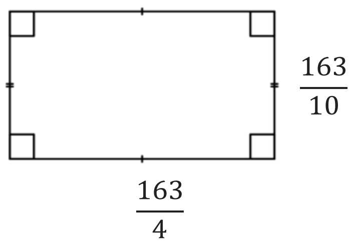
Section 8 – Topic 3
Introduction to Quadrilaterals – Part 3

1. Determine the area and perimeter of the quadrilaterals below.

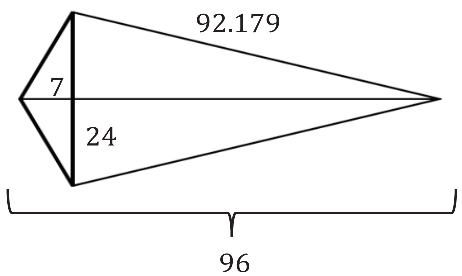
Part A:



Part B:

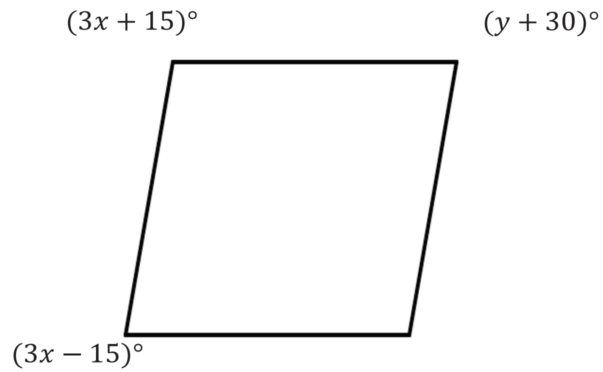


Part C:

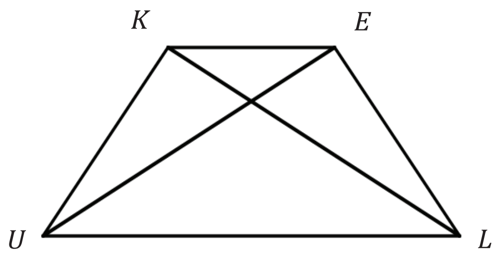


2. Determine the value of each variable for parallelogram $INDY$ that has diagonals that intersect at P. $IP = 3x, DP = 6x - 2,\ NP = 3y$, and $YP = 7x - 2$.

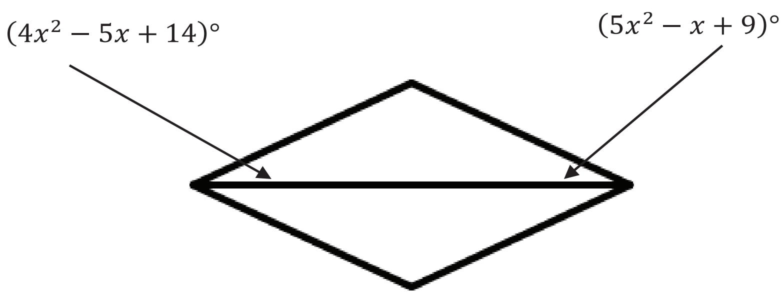
3. Determine the values of x and y for the parallelogram.



4. Determine the length of the diagonals of the isosceles trapezoid below when $KL = h + 7$ and $UE = 4h - 25$.

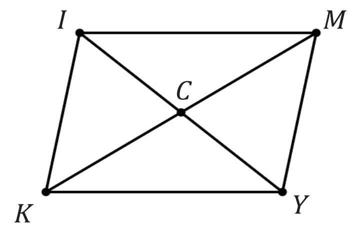


5. Find the value of x in the rhombus.



Section 8 – Topic 4
Parallelograms – Part 1

1. Consider the following parallelogram $IMYK$.



Determine which of the following are true statements. If the statement is false, then change the statement to be a true statement.

_______ $\Delta IKY \cong \Delta IMY$

_______ $m\angle IKY + m\angle KYM = 180°$

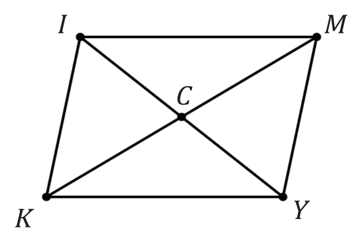
_______ $KY = IM$

_______ $\angle KIM \cong \angle KYM$

_______ $\Delta KMY \cong \Delta MKI$

_______ $m\angle MKY + m\angle MKI + m\angle KIY + m\angle YIM = 180°$

2. Consider the parallelogram $IMYK$ below.



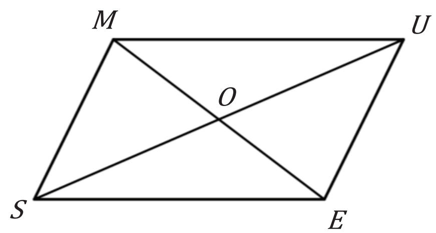
If $m\angle KIM = \frac{45}{7}x - \frac{2}{9}$ and $m\angle IKY = \frac{123}{7}x + \frac{20}{9}$, then determine the value of x to the nearest hundredth and find the measure of every angle in $IMKY$ above.

3. Consider $MUES$.

Given: Parallelogram $MUES$ with diagonals $\overline{SU}$ and $\overline{ME}$

Prove: $\Delta SOE \cong \Delta UOM$

Complete the following two-column proof.



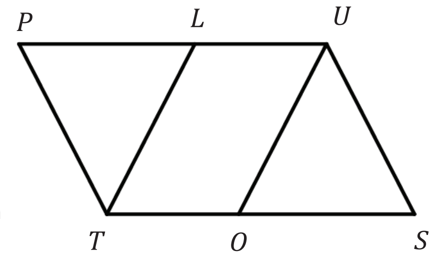
Statement	Reasons
1. Parallelogram $MUES$ with diagonals $\overline{SU}$ and $\overline{ME}$	**1.**
2. $\overline{MU} \cong \overline{SE}$ and $\overline{MS} \cong \overline{UE}$	**2.**
3. $\angle MSE \cong \angle MUE$ and $\angle SMU \cong \angle SEU$	**3.**
4. $\Delta SMU \cong \Delta UES$	**4.**
5. $\angle USE \cong \angle SUM$	**5.**
6. $\angle SEM \cong \angle UME$	**6.**
7. $\Delta SOE \cong \Delta UOM$	**7.**

4. Consider parallelogram $PUST$.

Given: Parallelogram $PUST$

$\overline{PL} \cong \overline{SO}$

Prove: $LUOT$ is a parallelogram



Complete the paragraph proof by filling in the justifications from the reasons bank.

Reasons Bank		
A. Adjacent Angles of a parallelogram are supplementary.	**B.** Definition of Angle Bisector	**C.** Diagonals of a parallelogram bisect each other.
D. Conditions of a parallelogram	**E.** Opposite sides of parallelograms are parallel.	**F.** Segment Addition Postulate
G. Subtraction Property of Equality	**H.** Angle Addition Postulate	**I.** Substitution
J. Opposite sides of a parallelogram are congruent.	**K.** Definition of Congruence	**L.** Definition of Segment Bisector

It is given that $\overline{PL} \cong \overline{SO}$ and $PUST$ is a parallelogram. It is known that $\overline{PU} || \overline{TS}$ because _____. We can say that $\overline{PU} \cong \overline{TS}$ because ______. We know that $PU = TS$ and $PL = SO$ because of the _______. Since $PU = PL + LU$ and $TS = TO + OS$ by the ______; then it is possible to say that $PL + LU = TO + OS$ by _______. The segments $LU = TO$ by the _____. Therefore $\overline{LU} \cong \overline{TO}$ because _____ and $LUOT$ is a parallelogram because of the ______.

Section 8 – Topic 5
Parallelograms – Part 2

1. The deck that Amos is building is in the shape of a parallelogram, $DGRY$. The measure of $\angle D$ is five-halves the measure of $\angle Y$. Find the measure of each angle of the deck.

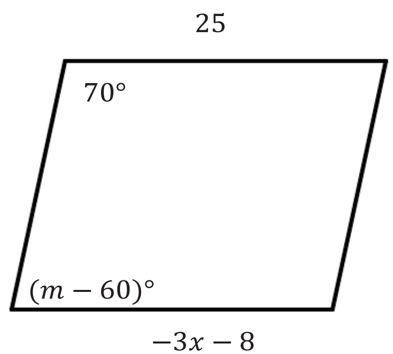
   $m\angle D =$

   $m\angle G =$

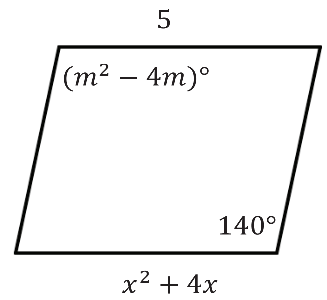
   $m\angle R =$

   $m\angle Y =$

2. Determine the values of the variables in the parallelogram below.



3. Determine the values of the variables in the parallelogram below.



4. Which set of information below can we use to conclude that quadrilateral $CATS$ can only be a parallelogram?

   Ⓐ $CA||ST$ and $SC = AT$

   Ⓑ $CA||ST$ and $AT||SC$

   Ⓒ $CA = AT$ and $SC = ST$

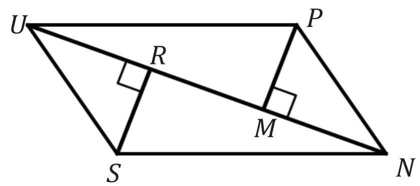
   Ⓓ $AS = TC$ and $\angle C + \angle T = 180$

5. Consider the figure below.

Given: Parallelogram $SUPN$, $\overline{SR} \perp \overline{UN}$ and $\overline{MP} \perp \overline{UN}$

Prove: $\Delta PMU \cong \Delta SRN$

Complete the two – column proof below.



Statement	Reasons
1. Parallelogram $SUPN$, $\overline{SR} \perp \overline{UN}$ and $\overline{MP} \perp \overline{UN}$	**1.** Given
2.	**2.** Definition of Perpendicular lines
3. $\angle SRU \cong \angle PMN$	**3.**
4.	**4.** Opposite sides of a parallelogram are parallel.
5.	**5.** Alternate Interior Angles are congruent.
6. $\overline{UP} \cong \overline{SN}$	**6.**
7. $\Delta PMU \cong \Delta SRN$	**7.**

Section 8 – Topic 6
Rectangles

1. The lengths of the diagonals of a rectangle are represented by $8x + 3$ feet and $4x + 7$ feet. Find the length of each diagonal.

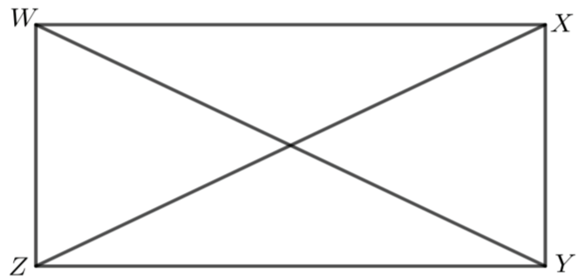
2. For rectangle $PLNE$, $m\angle PLE = 5x + 2$ and $m\angle NLE = 3x$. Determine the value of x, $m\angle PLE$, and $m\angle NLE$.

3. Rectangle $OPEN$ has diagonals intersecting at S. If $m\angle ENP = 42$, find $m\angle EON$, $m\angle ONP$, $m\angle OSP$, and $m\angle PSE$.

4. Complete the following proof.

Given: $WXYZ$ is a rectangle

Prove: $\overline{ZX} \cong \overline{WY}$

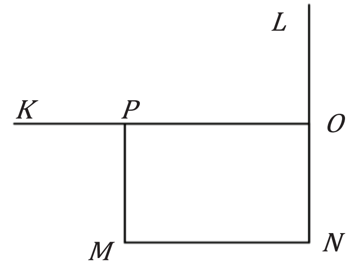


Statements	Reasons
1. $WXYZ$ is a rectangle.	**1.** Given
2. $WXYZ$ is a parallelogram.	**2.**
3.	**3.** Opposite sides of a parallelogram are congruent.
4. $\angle WZY$ and $\angle XYZ$ are right angles.	**4.**
5.	**5.** All right angles are congruent.
6. $\overline{ZY} \cong \overline{YZ}$	**6.**
7.	**7.** SAS Triangle Congruence Theorem
8. $\overline{ZX} \cong \overline{WY}$	**8.**

5. Complete the following proof.

Given: $MNOP$ is a parallelogram with $m\angle PMN = m\angle OPM$.

Prove: $MNOP$ is a rectangle.

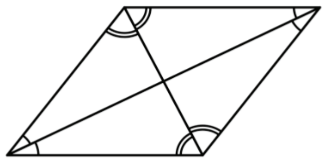


Statements	Reasons
1. $MNOP$ is a parallelogram with $m\angle PMN = m\angle OPM$.	**1.** Given
2. $m\angle PMN + m\angle OPM = 180°$	**2.**
3. $m\angle PMN + m\angle PMN = 180°$	**3.**
4. $2(m\angle PMN) = 180°$	**4.** Combining like terms
5. $m\angle PMN = 90°$	**5.**
6. $m\angle OPM = 90°$	**6.** Substitution
7. $\angle PMN \cong \angle NOP$ $\angle OPM \cong \angle MNO$	**7.**
8. $m\angle NOP = 90°; m\angle MNO = 90°$	**8.** Substitution
9.	**9.** Definition of a rectangle

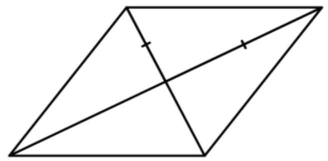
Section 8 – Topic 7
Rhombi

1. Determine if the following quadrilaterals are rhombi.

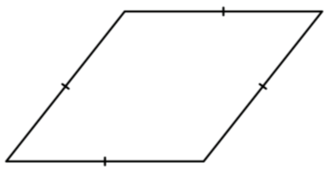
Part A:



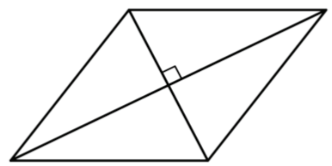
Part B:



Part C:



Part D:



2. A diagonal of a rhombus that is on the coordinate plane can be modeled by the equation $y = 6$. Determine which of the following could model the other diagonal. Justify your answer.

Ⓐ $y = 12$

Ⓑ $x = -6$

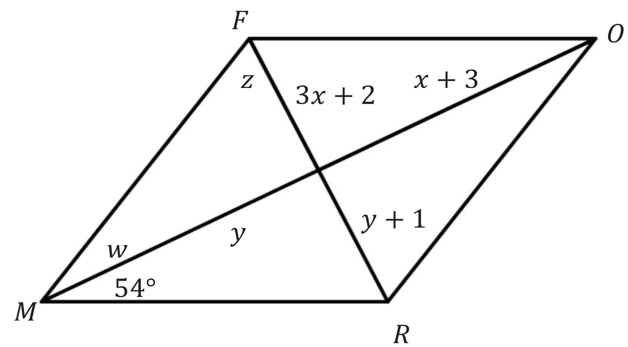
Ⓒ $y = 4 + x$

Ⓓ $x = 6 - y$

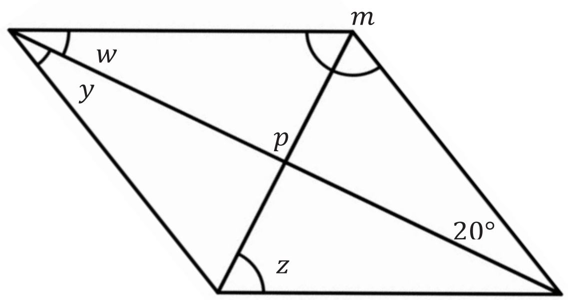
3. A rhombus is on a coordinate plane and has one of its sides modeled by the equation $3y - 4x = 35$. Determine which of the following equations could model the opposite side of the rhombus. Select all that apply. Justify your answer.

- ☐ $y = \frac{4}{3}x + \frac{16}{3}$
- ☐ $y - 3 = \frac{3}{4}(x - 8)$
- ☐ $8x - 6y = 38$
- ☐ $9y = 12x - \frac{27}{14}$
- ☐ $2x + y = -5$
- ☐ $4x - 3y = 12$

4. Determine the value of every variable in the rhombus below.



5. Determine the value of every variable in the rhombus below.



6. The perimeter of a rhombus is 560 meters and one of its diagonals has a length of 76 meters. Find the area of the rhombus.

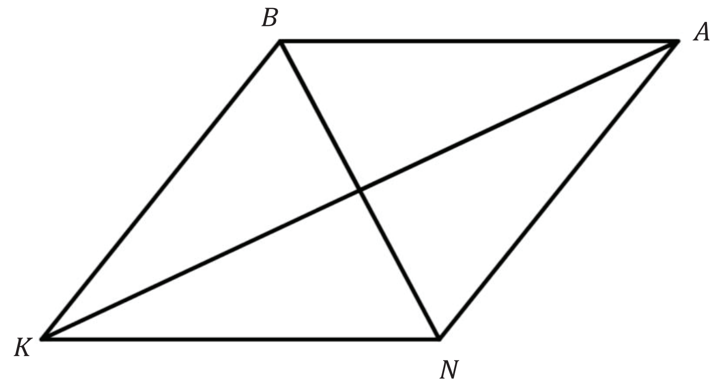
Part A: Determine the length of the other diagonal.

Part B: The area is ______________________ square meters.

7. Consider the following information.

Given: $BANK$ is a parallelogram with $KB = 10x - 10, AN = 6x + 6$, and $BA = \frac{15}{2}x$.

Prove: $BANK$ is a rhombus.



Complete the following paragraph proof.

$BANK$ is a parallelogram with $KB = 10x - 10, AN = 6x + 6$, and $BA = \frac{15}{2}x$.

Because in a parallelogram opposite sides are ______________________, $\overline{KB} \cong \overline{AN}$, and _______ $\cong$ _______. Therefore, $6x + 6 = 10x - 10$. By ____, $-4x = -16$. By Division Property of Equality, $x =$ _______. By substitution, $NK = AN =$ _______ units, and $BA = NK =$ _______ units. $BANK$ is a rhombus, because ________________________________.

Section 8 – Topic 8
Squares

1. Given two segments with lengths M and N, where $M \neq N$, describe and sketch a parallelogram that meets the given scenario.

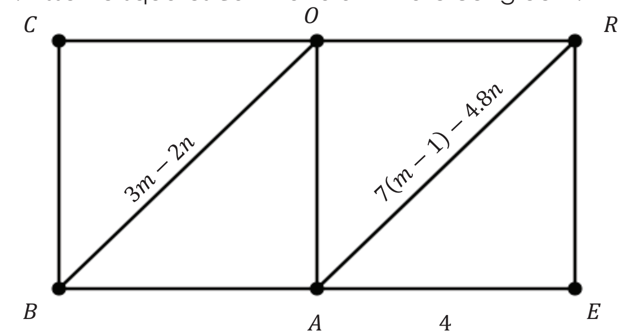
   Part A: Both diagonals have length M and are perpendicular.

   Part B: Both diagonals have length N but are not perpendicular.

   Part C: One diagonal has length M, the other diagonal has length N, and they meet at a right angle.

2. Shawna cuts out a square piece of cardstock paper for a project. Her partner, Tanisha wants to check to see if the cutout is actually a square. Tanisha checks to see if the diagonals are congruent. Does this determine if the piece of cardstock paper is an actual square? Justify your answer.

3. Consider quadrilateral $CREB$ made up of squares $COAB$ and $OREA$, where $\overline{AE} = 4$. Assume squares $COAB$ and $OREA$ are congruent.



   Determine the values of m and n and the length of $\overline{RB}$ in the quadrilateral $CREB$.

4. Consider the following scenarios using two strings.

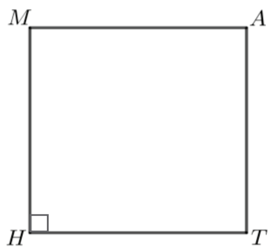
   Part A: Is it possible to construct a square with the two strings? Explain your answer.

   Part B: Is it possible to construct a rectangle with the two strings? Explain your answer.

5. Given that $\overline{MA} \cong \overline{HT}$, $\overline{MH} \cong \overline{AT}$, $\overline{MA} \cong \overline{AT}$, and $\angle H$ is a right angle, show that $MATH$ is a square using a paragraph proof.

Given: $\overline{MA} \cong \overline{HT}$, $\overline{MH} \cong \overline{AT}$, $\overline{MA} \cong \overline{AT}$, $\angle H$ is a right angle

Prove: $MATH$ is a square

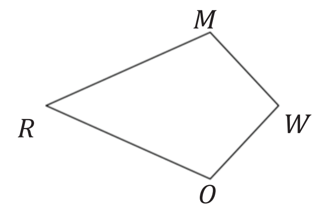


Section 8 – Topic 9
Kites

1. Use the Reasons Bank's abbreviations to complete the two-column proof.

Given: Quadrilateral WORM is a kite.

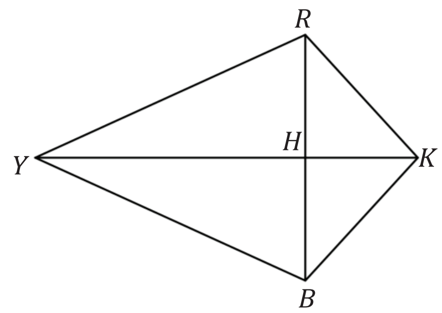
Prove: $\overline{MO} \perp \overline{RW}$.



Reasons Bank		
Congruent (C)	Definition of Kite (DK)	All perpendicular lines are congruent (APLC).
Definition of Perpendicular Bisector (DPB)	Angle Bisector (AB)	Converse of Perpendicular Bisector Theorem. (CPBT)
Two points determine a line (TPL).	Isosceles Triangle(IT)	Opposite Angles (OA)

Statements	Reasons
1. Quadrilateral $WORM$ is a kite.	**1.** Given
2. $\overline{MW} \cong \overline{OW}$ and $\overline{RM} \cong \overline{RO}$	**2.**
3. R and W lie on a perpendicular bisector of $\overline{MO}$.	**3.**
4. $\overline{RW}$ is a perpendicular bisector of $\overline{MO}$.	**4.**
5. $\overline{MO} \perp \overline{RW}$	**5.**

2. Consider kite $BKRY$ below where $m\angle BKH = 36$.



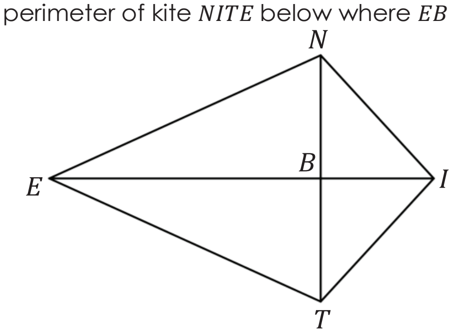
Part A: Determine $m\angle BHY$. Justify your answer.

Part B: Determine $m\angle RKH$. Justify your answer.

Part C: Determine $m\angle HBK$. Justify your answer.

Part D: Determine $m\angle RYB$. Justify your answer.

3. Determine the perimeter of kite $NITE$ below where $EB = 84$ feet and $BI = BN = 35$.



4. Determine if the following statements are true (T) or false (F).

____ A kite can have opposite angles that are both supplementary.

____ A kite can have consecutive angles that are supplementary.

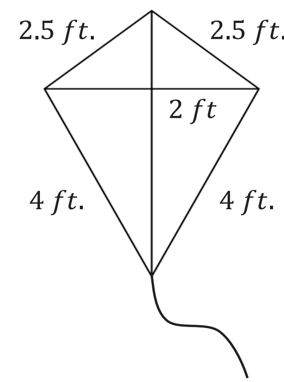
____ A kite can have opposite angles that are acute.

____ A kite can have consecutive angles that are obtuse.

____ A kite can have opposite angles that are complementary.

____ A kite can have consecutive angles that are complementary.

5. Mary and Bert make a kite for the festival that is next Saturday. Their kite and measurements are below.



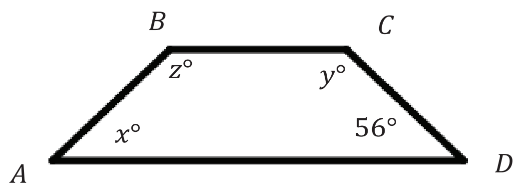
Part A: Complete the blanks of the paragraph.

Mary and Bert's kite fits the geometric definition of a kite because it has two pairs of congruent _____________ sides and no pairs of congruent _____________ sides. The vertical and horizontal supports of the kite are its _____________. The vertical support _____________ the horizontal support. Because the diagonals of a kite are _____________ to each other, they divide the kite into four _____________ triangles. The kite's vertical support divides it into two _____________ triangles.

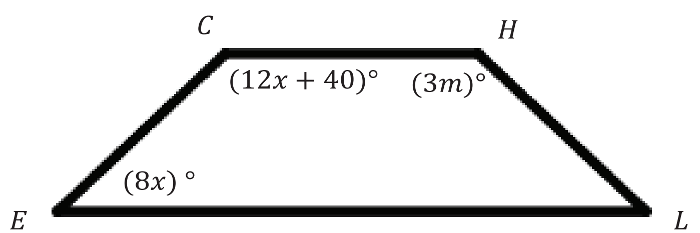
Part B: The festival manager needs the support measurements for every kite to be flown next Saturday. Determine the lengths of each support.

Section 8 – Topic 10
Trapezoids

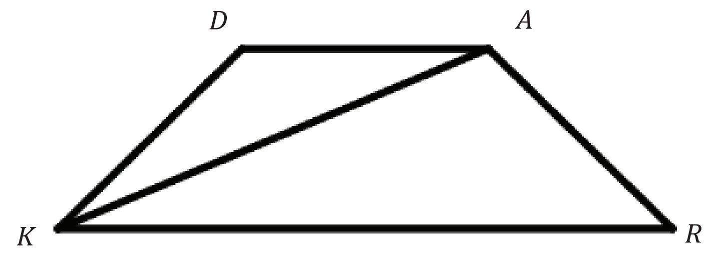
1. Determine the values of the variables in the figure below, where $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \parallel \overline{DA}$.



2. Determine the values of the variables in the isosceles trapezoid $CHLE$ below.



3. Prove that quadrilateral $DARK$ is a trapezoid, given that $\overline{KA}$ bisects $\angle DKR$, $\overline{DK} \cong \overline{RA}$, and $\overline{DA} \parallel \overline{KR}$.

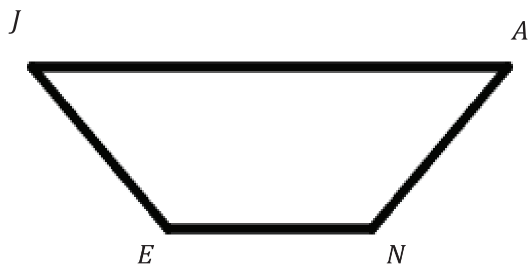


Which of the following reasons can be used to justify the steps that are necessary to prove that quadrilateral $DARK$ is a trapezoid? Select all that apply.

- ☐ Alternate Interior Angles Theorem
- ☐ Angle bisector
- ☐ Alternate Exterior Angles Theorem
- ☐ Definition of a parallelogram
- ☐ Same side / Consecutive Interior Angles
- ☐ Transitive Property
- ☐ Pythagorean Theorem

4. Isosceles trapezoid $SAND$ has diagonals $SN = 5w + 13$ and $AD = 11w - 5$. Determine the value of w.

5. Consider trapezoid $JANE$ with bases $\overline{JA}$ and $\overline{EN}$.



Complete the following statements and reasons.

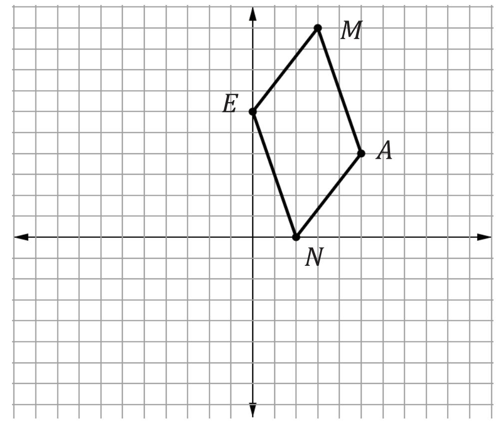
Statement	Reason
1. $\overline{JA} \parallel$ ______	**1.** Definition of trapezoid
2. $\angle J$ and ______ are supplementary.	**2.**
3. $\angle N$ and ______ are supplementary.	**3.**

Section 8 – Topic 11
Quadrilaterals in Coordinate Geometry – Part 1

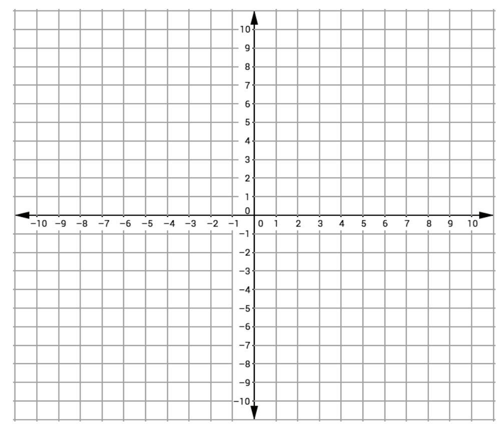
1. Consider the information and figure below.

Given: $MANE$ is a quadrilateral with vertices $M\ (6,20)$, $A(10,8)$, $N(4,0)$, and $E\ (0,12)$.

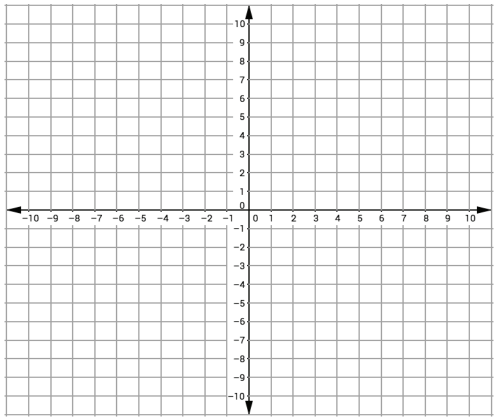
Prove: Quadrilateral $MANE$ is a parallelogram.



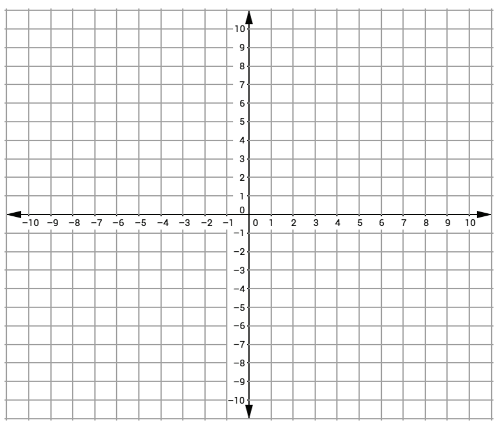
2. Consider a quadrilateral with vertices $D(0,0)$, $I(5,5)$, $N(8,4)$, and $G(7,1)$. Determine the most accurate type of quadrilateral from the vertices given.



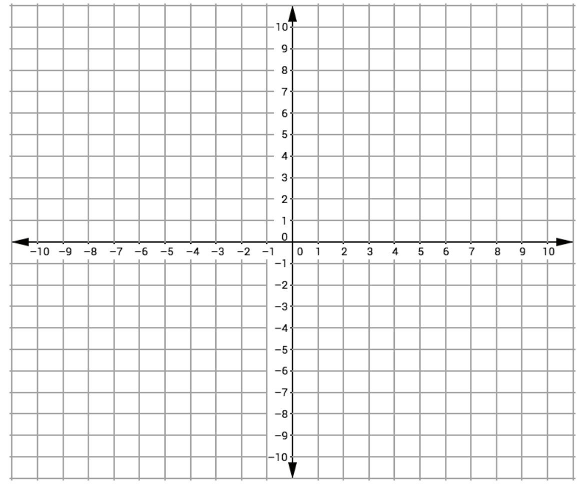
3. Consider a quadrilateral with vertices $H(2,1)$, $A(5,4)$, $M(8,1)$, and $S(2,-3)$. Determine the most accurate type of quadrilateral from the vertices given.



4. Consider a quadrilateral with vertices $S(-7,2)$, $Q(0.5,3)$, $R\left(\frac{3}{2},-\frac{9}{2}\right)$, and $E(-6,-5.5)$. Determine the most accurate type of quadrilateral from the vertices given.



5. The diagonals of a quadrilateral intersect at $(-1,4)$. One of the sides of the quadrilateral is bounded by $(2,7)$ and $(-3,5)$.



Part A: Determine the coordinates of the other side in order for the quadrilateral to be a parallelogram.

Part B: Determine the coordinates of the other side in order for the quadrilateral to be a square.

Section 8 – Topic 12
Quadrilaterals in Coordinate Geometry – Part 2

1. A square has the vertices at $(-2, 6), (6, 1), (1, -7)$, and $(-7, -2)$. At what point do the diagonals of the square intersect?

2. A rhombus, $COAL$ is centered at the origin. The longer diagonal is on the y-axis and has a length of $27m$. The shorter diagonal is on the x-axis and has a length of $19m$. Determine the coordinates of the vertices.

3. Determine the most precise name of the quadrilateral with the vertices at $H(b, 2c)$, $O(4b, 3c), M(5b, c)$, and $E(2b, 0)$.

4. Explain how each scenario can be proven using coordinate geometry.

   Part A: The diagonals of a rhombus are each other's bisectors.

   Part B: A parallelogram is formed when the midpoints of the sides of an isosceles trapezoid are connected.

5. Determine which of the following formula(s) are needed to prove that a quadrilateral is an isosceles trapezoid. Select all that apply.

   ☐ Midpoint Formula

   ☐ Slope Formula

   ☐ Distance Formula

   ☐ The Area Formula

   ☐ The Perimeter Formula

Section 9: Circles – Part 1
Student Learning Plan

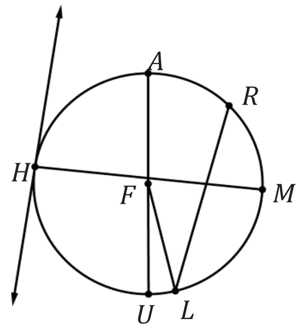
Topic Number	Topic Name	Date Completed	Study Expert(s)	Check Your Understanding Score
1	Circumference of a Circle			
2	Arcs and Circumference of a Circle			
3	Area of a Circle			
4	Sectors of a Circle			
5	Circles in the Coordinate Plane: Standard Form			
6	Circles in the Coordinate Plane: General Form			
7	Circle Transformations			
8	Radians and Degrees			

What did you learn in this section? What questions do you still have?

Who was your favorite Study Expert for this section? Why?

Section 9 – Topic 1
Circumference of a Circle

1. Consider the following figure and answer the questions below it.



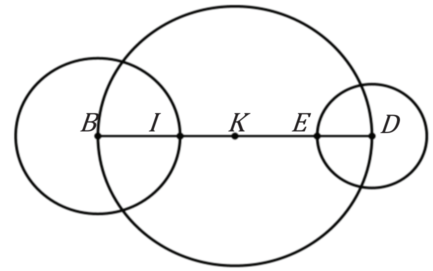
Part A: Identify the parts of the circle in each space provided below.

Center	
Radius	
Chord	
Diameter	

Part B: If the diameter of circle F is 14 inches, then determine the radius length of circle F.

2. For circle R, with point L lying on the circle, determine the circumference if $\overline{RL} = 34$ units.

3. The radii of circles B, K, and D are 6 feet, 14 feet, and 3 feet, respectively.

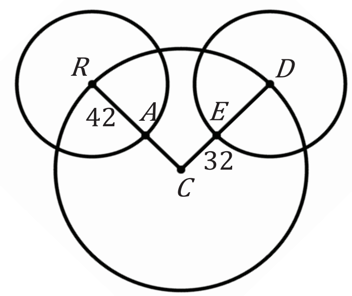


Part A: Determine the length of $\overline{IK}$. Justify your answer.

Part B: Determine the length of $\overline{EK}$. Justify your answer.

Part C: Determine the length of $\overline{BE}$. Justify your answer.

4. Consider the figure below with circles R, C, and D below, where circle R and circle D are congruent. If $\overline{RA} = 42$ and $\overline{CE} = 32$, then answer the questions below.

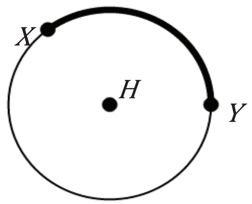


Part A: Determine $CA + ED$.

Part B: Determine the combined circumference of all three circles.

Section 9 – Topic 2
Arcs and Circumference of a Circle

1. Consider circle H below.



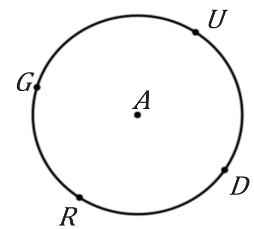
Part A: If $\overline{XH} = 10$ feet and $m\widehat{XY} = 100°$, then determine the arc length of $\widehat{XY}$.

Part B: If $\overline{HY} = 46$ inches and $m\widehat{YX} = 75°$, then determine the arc length of $\widehat{YX}$.

Part C: If $\widehat{YX} = 24$ meters and $m\widehat{YX} = 120°$, then determine the radius of circle H.

Part D: If $\widehat{XY} = 78$ miles and $m\widehat{XY} = 70°$, then determine the radius of circle H.

2. Consider the circle below with center A.

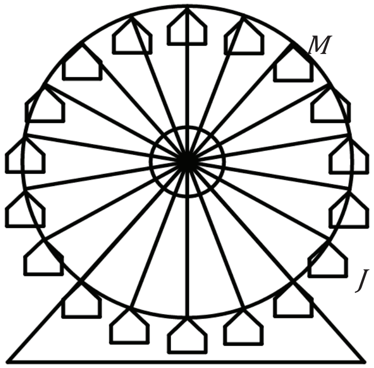


Part A: If $\overline{GA} = 12$ feet and a major arc $m\widehat{GR} = 200°$, then determine the major arc length of $\widehat{GR}$.

Part B: If $\overline{GA} = 29$ and a major arc $m\widehat{DG} = 185°$, then determine the minor arc length of $\widehat{GD}$.

Part C: If the minor arc $\widehat{GU} = 24$ units and major arc $m\widehat{GU} = 270°$, then determine the radius of circle A.

3. The *Skyview Atlanta* in Atlanta, Georgia is a super-sized Ferris wheel that overlooks the city and is 200 feet in radius length.

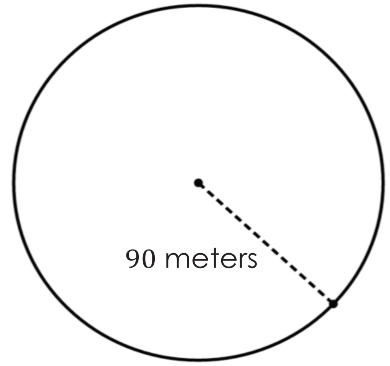


Part A: If a passenger rides clockwise from points M to point J and then stops, determine how many feet the passenger has traveled.

Part B: Approximate, to the nearest foot, how many feet a passenger would travel if the full ride is two revolutions.

Section 9 – Topic 3
Area of a Circle

1. Consider the circle below with radius given.



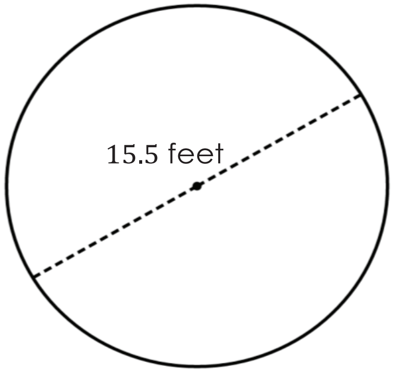
Part A: Determine the diameter of the circle above.

Part B: Determine the circumference of the circle above, leave in terms of π.

Part C: Determine the exact area of the circle above.

2. Michelle Obama helped design the $3{,}520$ – piece china set that was used at the White House. The ten - inch dinner service plate was first used for the then – current Japanese Prime Minister. Determine the area of the ten–inch service plate.

3. Consider the circle below with diameter given. Round all answers to the nearest hundredth.



Part A: Determine the radius of the circle above.

Part B: Determine the circumference of the circle above.

Part C: Determine the area of the circle above.

4. What is the area of a circle whose circumference is 8π?

   Ⓐ 12π
   Ⓑ 8π
   Ⓒ 16π
   Ⓓ 4π

5. If a circumference of a circle is 10π inches, then determine the area of the circle.

   Ⓐ 10π square inches
   Ⓑ 25π square inches
   Ⓒ 50π square inches
   Ⓓ 100π square inches

6. An asteroid hit the Earth and created a huge crater. Engineers and scientists have measured around the crater and found it to be 79.5 miles. How much of the Earth's surface was affected by the crater impact?

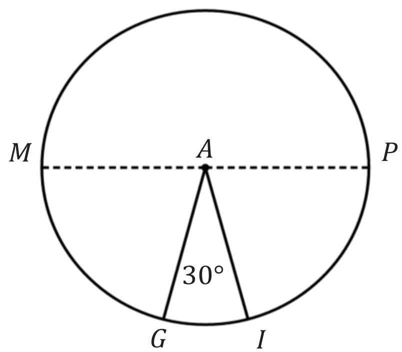
7. A circular city park has a sidewalk directly through its middle that is 111 – feet long. If each bag of fertilizer covers 50 square feet, then determine how many bags of fertilizer the Parks and Recreation department needs to cover the circular park. *Ignore all the sidewalks around and through the park*.

Section 9 – Topic 4
Sectors of a Circle

1. A circle has an 18-inch radius and a shaded sector with a central angle of $50°$. Determine the area of the shaded sector.

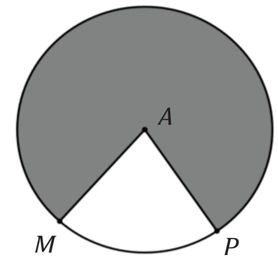
2. The area of a sector with a radius of 14 yards is 38.28 square yards. Calculate the approximate angle of the sector. Round to the nearest tenth.

3. In the diagram below of circle A, diameter $MP = 26$, $m\angle GAI = 30°$ and radii $\overline{GA}$ and $\overline{AI}$ are drawn.

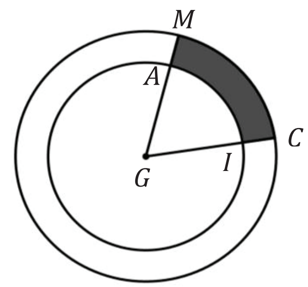


If $\widehat{MG} \cong \widehat{IP}$, find the area of the sector MAG in terms of π and approximate to the nearest hundredth.

4. Determine the area of the shaded sector in the circle below with the measure of major arc $\widehat{MP} = 280°$ and $MA = 32$ inches.



5. *PaintsPlus LLC* specializes in circular paint jobs. Their most recent job is modeled in the diagram below.

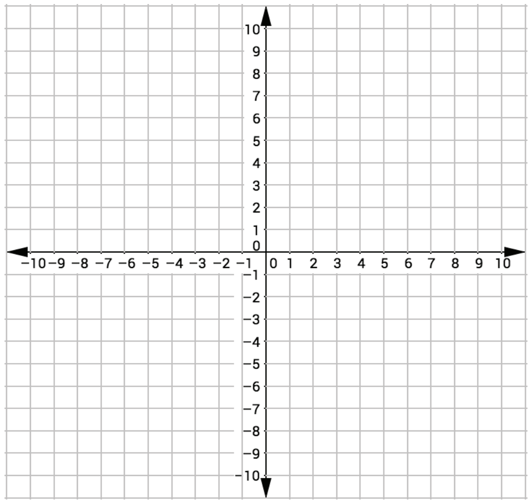


The two circles have center G, where radius $GI = 4$ feet, radius $MG = 6.5$ feet, and $m\angle MGC = 72°$. Determine the total cost to paint area $MCAI$ if the quoted price is \$40 per square foot. Leave your answers in terms of π until calculating the cost and then round to the nearest dollar.

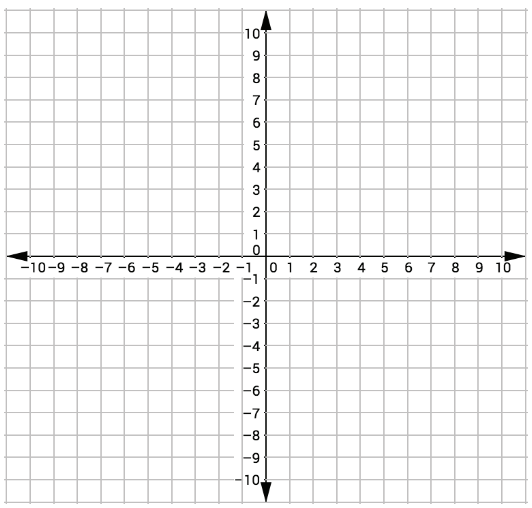
Ⓐ $670
Ⓑ $680
Ⓒ $660
Ⓓ $620

Section 9 – Topic 5
Circles in the Coordinate Plane: Standard Form

1. Graph the circle from the standard form $(x-1)^2 + y^2 = 49$.

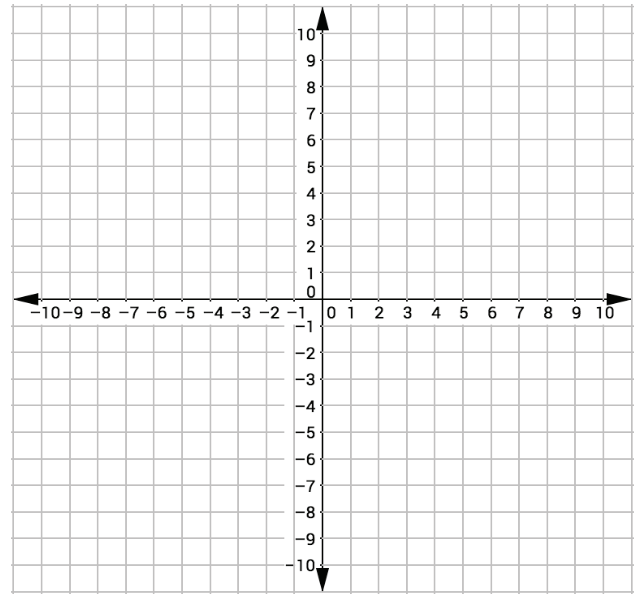


2. Graph the circle from the standard form $(x+4)^2 + (y+2)^2 = 25$.



3. Mr. Iwan wanted his students to graph the following circles on the same coordinate plane.

Circle 1: $x^2 + (y+4)^2 = 36$
Circle 2: $(x-5)^2 + (y-4)^2 = 16$
Circle 3: $(x+5)^2 + (y-4)^2 = 16$



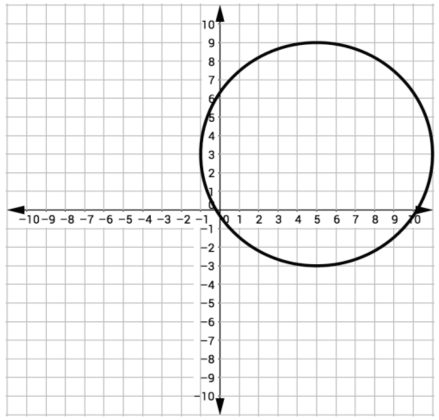
Part A: Graph the three circles on the coordinate plane above.

Part B: One of Mr. Iwan's students, Bret, wanted to add some more circles to the already plotted three and decided a circle should be centered $(-2, -2)$ with a radius of one unit. Graph Bret's new circle.

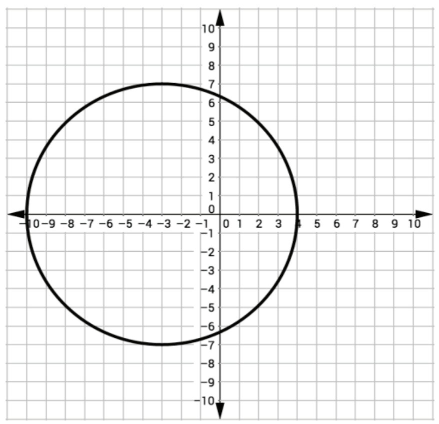
Part C: Write the equation of Bret's new circle.

Part D: Mr. Iwan was a great grader and decided to mark another circle on Bret's design that was reflected on the y −axis to make the design look like a celebrity. Write the equation of Mr. Iwan's circle and graph Mr. Iwan's circle above.

4. Write the equation of the circle in standard form from the following graph.



5. Write the equation of the circle in standard form from the following graph.



6. The local cell phone tower used by your cell phone provider has the best signal, or "bars," when you are 15 miles from the tower.

Part A: If you are 12 miles east and 11 miles south of the cell tower, will you receive the most bars? Justify your answer.

Part B: If you are 13 miles west and 6 miles north of the cell tower, then will you receive the most bars? Justify your answer.

Section 9 – Topic 6
Circles in the Coordinate Plane: General Form

1. Complete the square to transform the general form equation below to standard form in order to determine the center and radius.

$$4x^2 + 4y^2 - 16x - 24y + 51 = 0$$

The center is at (__________, __________) and the radius is _______ units.

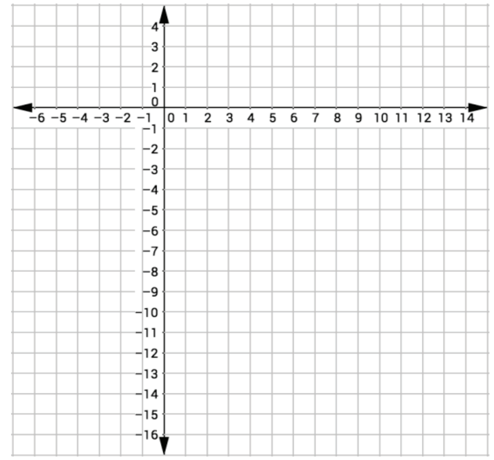
2. Complete the square to transform the general form equation below to standard form in order to determine the center and radius.

$$x^2 + y^2 + 14x + 18y + 114 = 0$$

The center is at (__________, __________) and the radius is _______ units.

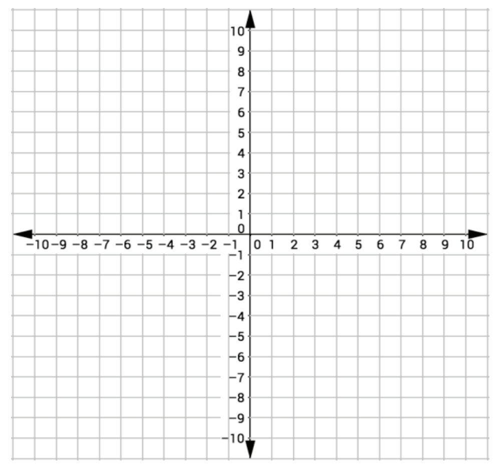
3. Complete the square to transform the general form equation below to standard form in order to determine the center and radius, then graph the circle.

$$x^2 + y^2 - 4x + 22y + 116 = 0$$



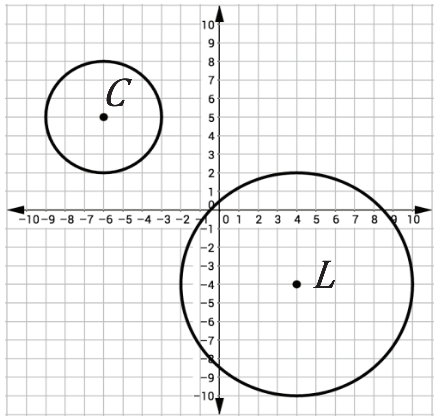
4. Complete the square to transform the general form equation below to standard form in order to determine the center and radius, then graph the circle.

$$x^2 + y^2 + 6x + 10y + 18 = 0$$



Section 9 – Topic 7
Circle Transformations

1. Consider the two circles on the coordinate plane below.



Part A: Describe the transformation from circle C to circle L.

Part B: Describe the transformation from circle L to circle C.

Part C: Compare and contrast the standard form of the equation for both circles above.

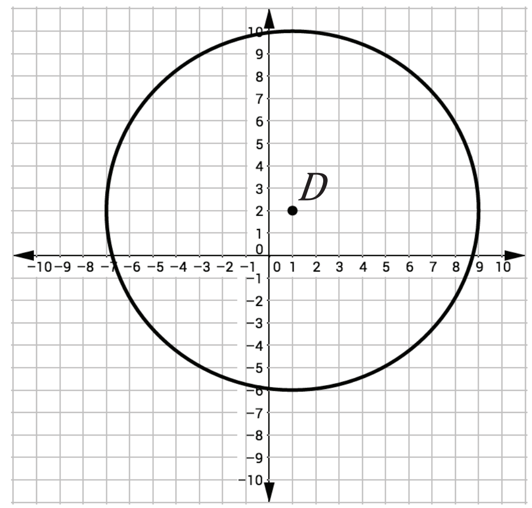
Part D: Graph the result of a transformation of circle L using the rule $(x, y) \rightarrow (x - 4, y + 4)$ followed by a dilation of a scale factor of $\frac{3}{2}$ centered at point L'.

2. Circle G and circle H have their standard form of the equation below.

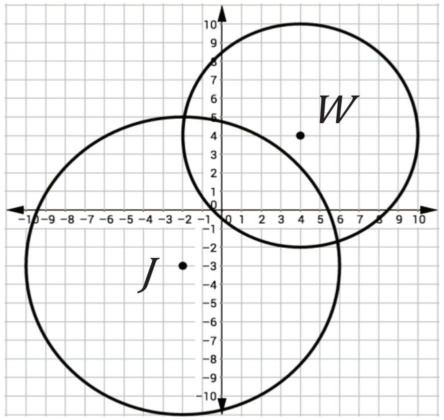
Circle G: $(x - 14)^2 + (y + 12)^2 = 49$
Circle H: $(x - 18)^2 + (y - 8)^2 = 196$

Describe the transformation from circle G to circle H.

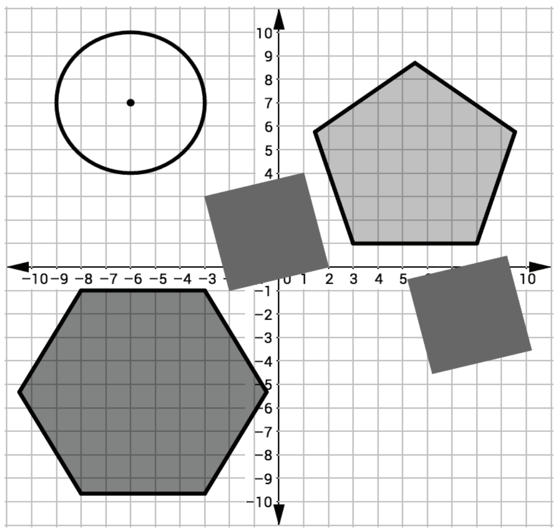
3. Graph the result of a transformation using the rule $(x, y) \rightarrow (x - 3, y - 4)$ followed by a dilation of scale factor $\frac{3}{8}$ centered at D' in the coordinate plane below.



4. Describe the sequence of transformations that carry circle W to circle J.



5. Allison is creating an abstract piece of art, or cubism, using only geometric shapes. The model of her artwork is below.



Allison wants to transform her circle into the blank space between the hexagon and the bottom square. Describe the transformation(s) that allow the circle to fill the most space between the two shapes.

Section 9 – Topic 8
Radians and Degrees

1. Convert the following angle measurements to radians.

Part A: $150°$

Part B: $220°$

Part C: $40°$

Part D: $520°$

2. Convert the following radian measure into degrees.

Part A: $\frac{5\pi}{4}$

Part B: $\frac{2\pi}{5}$

Part C: $\frac{14\pi}{15}$

Part D: $\frac{7\pi}{3}$

3. What is the length of the arc with a measure of $100°$ in a circle with a radius of 15 inches?

4. An arc with a measure of $190°$ has an arc length of 40π centimeters. What is the radius of the circle on which the arc sits?

5. An arc has a length of 123.53 millimeters and a radius of 28 millimeters. What is the angle of the sector in radians?

6. What is the length of the arc with a measure of $200°$ in a circle with a radius of 31 inches?

Section 10: Circles – Part 2
Student Learning Plan

Topic Number	Topic Name	Date Completed	Study Expert(s)	Check Your Understanding Score
1	Arcs and Inscribed Angles – Part 1			
2	Arcs and Inscribed Angles – Part 2			
3	Inscribed Polygons in a Circle			
4	Constructing Polygons Inscribed in a Circle			
5	Tangent Lines, Secants, and Chords – Part 1			
6	Tangent Lines, Secants, and Chords – Part 2			
7	Circumscribed Angles and Beyond – Part 1			
8	Circumscribed Angles and Beyond – Part 2			
9	Constructing Inscribed and Circumscribed Circles of Triangles			
Honors 1	Tangent Lines Through an External Point of a Circle			

**Honors resources are available online.*

What did you learn in this section? What questions do you still have?

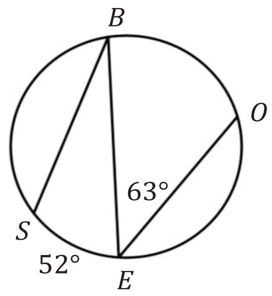
Who was your favorite Study Expert for this section? Why?

Section 10 – Topic 1
Arcs and Inscribed Angles – Part 1

1. Consider the circle to the right.

   Part A: Determine $m\angle SBE$.

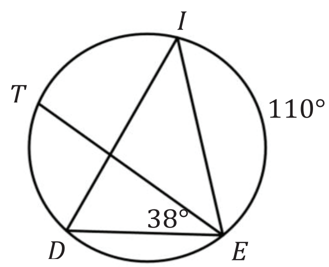
   Part B: Determine $m\widehat{OB}$.



2. Consider the circle to the right.

   Part A: Determine $m\angle IDE$.

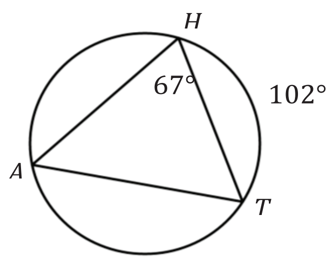
   Part B: Determine $m\widehat{TD}$.



3. Consider the circle to the right.

   What is $m\angle HTA$?

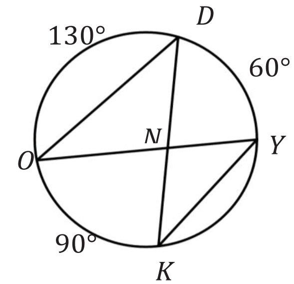
   Ⓐ 51°
   Ⓑ 67°
   Ⓒ 62°
   Ⓓ 102°



4. Consider the circle to the right.

   Determine $m\angle KNY$.

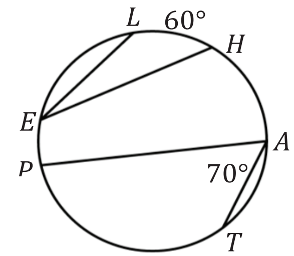
   Ⓐ 30°
   Ⓑ 45°
   Ⓒ 65°
   Ⓓ 105°



5. Consider the circle to the right.

   Part A: Determine $m\angle LEH$.

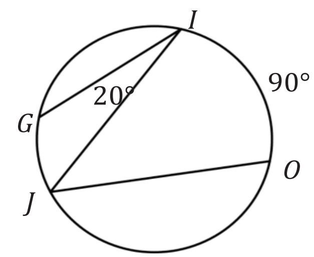
   Part B: Determine $m\widehat{PT}$.



6. Consider the circle to the right.

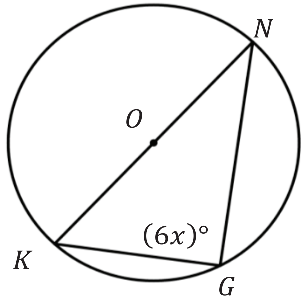
   Part A: Determine $m\angle IJO$.

   Part B: Determine $m\widehat{GJ}$.



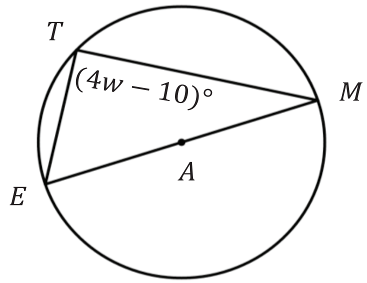
7. Consider the circle to the right.

Determine the value of x.



8. Consider the circle to the right.

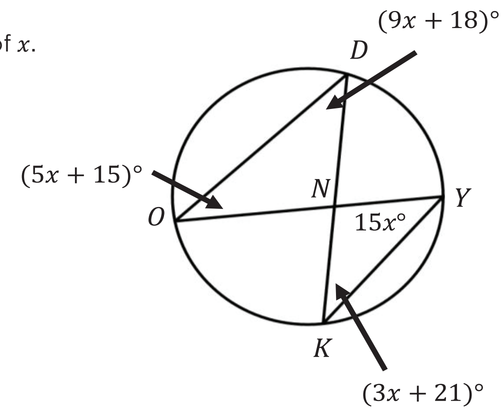
Determine the value of w.



Section 10 – Topic 2
Arcs and Inscribed Angles – Part 2

1. Consider the circle on the right.

Part A: Determine the value of x.



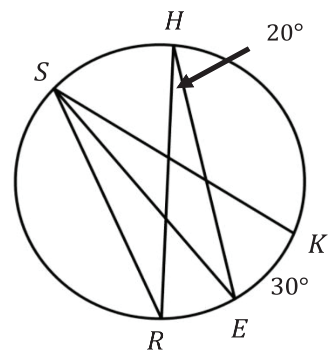
Part B: Determine $m\angle YNK$.

Ⓐ $105°$
Ⓑ $122.5°$
Ⓒ $126°$
Ⓓ $120°$

2. Consider the circle on the right, where $m\angle RHE = 20°$ and $m\widehat{EK} = 30°$

Determine $m\angle RSK$.

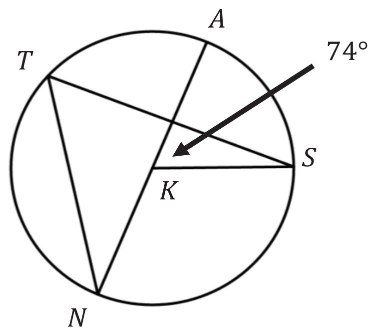
Ⓐ $25°$
Ⓑ $35°$
Ⓒ $50°$
Ⓓ $70°$



3. Consider circle K on the right, where $m\angle SKA = 74°$.

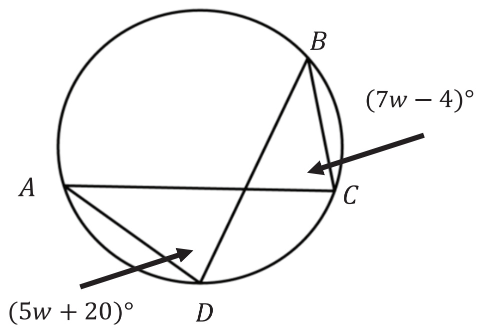
Determine $m\angle NTS$.

Ⓐ 37°
Ⓑ 74°
Ⓒ 53°
Ⓓ 106°



4. Consider the circle on the right.

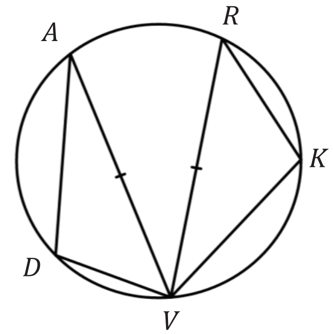
Determine $m\widehat{AB}$.



5. Consider the following figure where $\overline{VA} \cong \overline{RV}$.

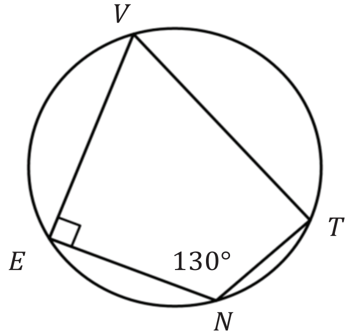
Which one of the following statements is correct?

Ⓐ $\angle VDA \cong \angle VKR$
Ⓑ $\widehat{RKV} = \widehat{ARK}$
Ⓒ $m\angle VKR = \widehat{VKR}$
Ⓓ $\angle DAV \cong \angle KRV$



Section 10 – Topic 3
Inscribed Polygons in a Circle

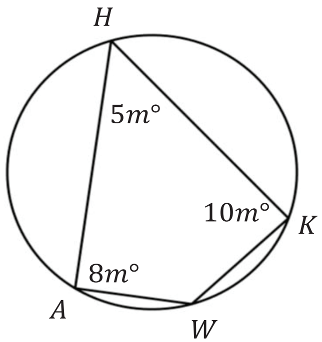
1. Consider the circle to the right.



Part A: Determine $m\angle T$.

Part B: Determine $m\angle V$.

2. Consider the circle to the right.



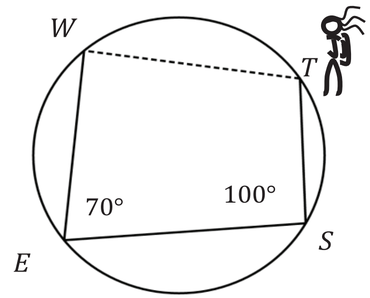
Part A: Determine $m\angle H$.

Part B: Determine $m\angle A$.

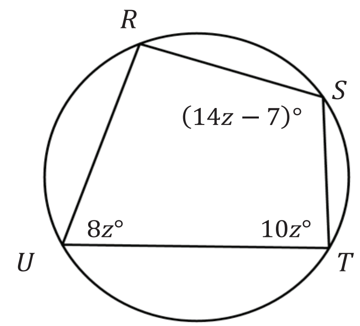
Part C: Determine $m\angle W$.

Part D: Determine $m\angle K$.

3. Ellen "The Wonderer" continues to walk on the path of an inscribed polygon. She starts at W, walks south until the boundary point E, turns $70°$ and walks east until S, turns $100°$, and walks north to point T. Determine how many degrees Ellen must turn to get back to her starting point.



4. Consider the circle to the right.



Part A: Determine the value of z.

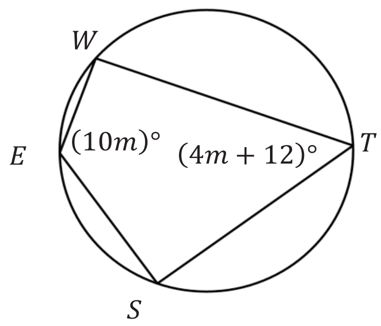
Part B: Determine $m\angle U$.

Part C: Determine $m\angle S$.

Part D: Determine $m\angle R$.

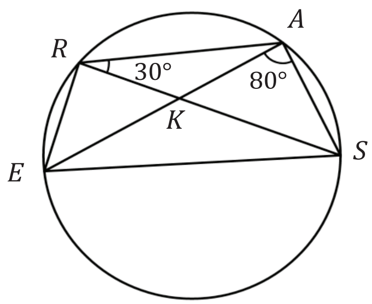
5. Consider the circle to the right.

   Part A: Determine $m\angle E$.



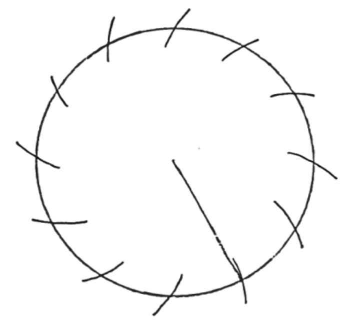
   Part B: Determine $m\angle T$.

6. Determine $m\angle ESA$ in the circle below.

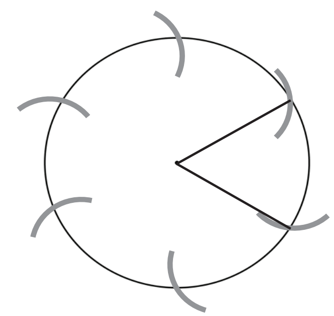


Section 10 – Topic 4
Constructing Polygons Inscribed in a Circle

1. Determine which regular polygons can be constructed from the construction below.



2. Consider the process of construction below.

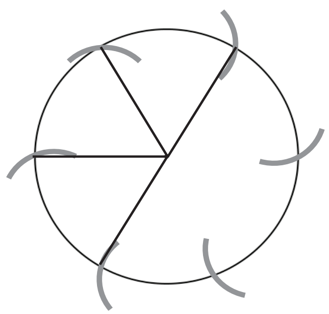


   Part A: Describe how to construct an equilateral triangle.

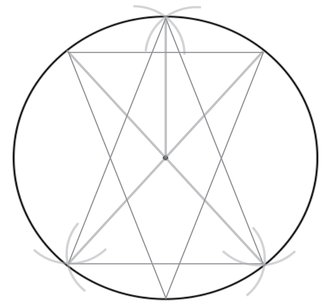
   Part B: Describe how to construct a regular hexagon.

Part C: Determine if it is possible to construct a square using the construction above. Justify your answer.

3. Bartholomew wanted to construct an octagon. He started the process, but his brother Matthew informed Bartholomew that he was wrong. How could Matthew help his brother fix his mistakes in order to draw an octagon?



4. Consider the construction below.

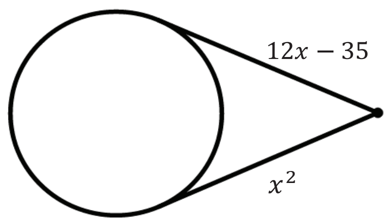


Determine the polygon that could be constructed from the isosceles triangles.

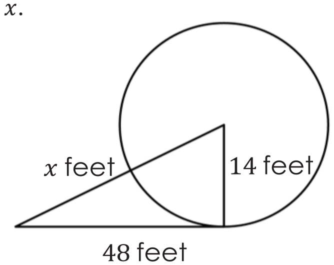
5. Construct a square inscribed in circle C below.

Section 10 – Topic 5
Tangent Lines, Secants, and Chords – Part 1

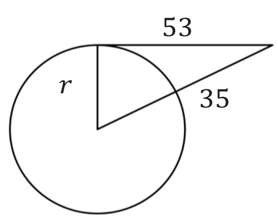
1. Determine the possible value(s) of x.



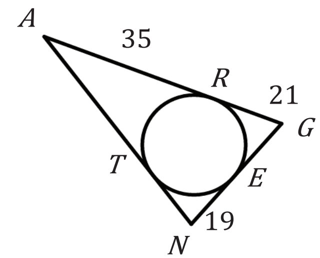
2. Determine the value of x.



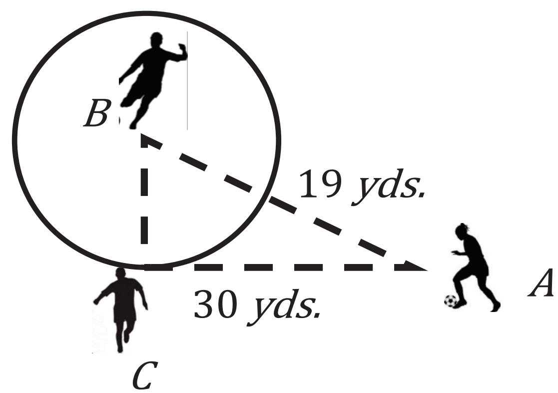
3. Determine the radius of the circle below.



4. Determine the perimeter of triangle AGN.

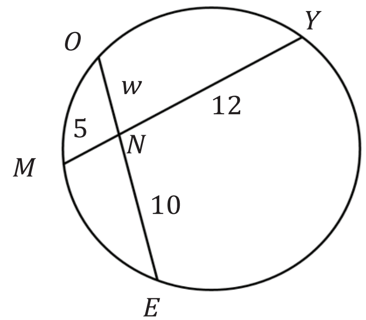


5. Boyd (B) is trapped in a soccer training circle. Albert (A) and Carlos (C) kick the ball to Boyd in rotation. Determine how far Boyd needs to kick the ball to Carlos if he is located in the center of his training circle and Carlos is located at the outer rim.

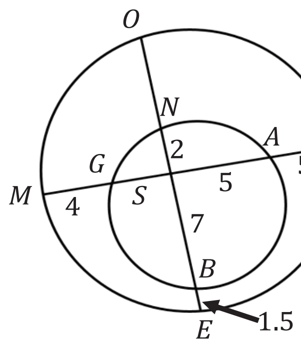


Section 10 – Topic 6
Tangent Lines, Secants, and Chords – Part 2

1. Determine the value of w in the circle below.



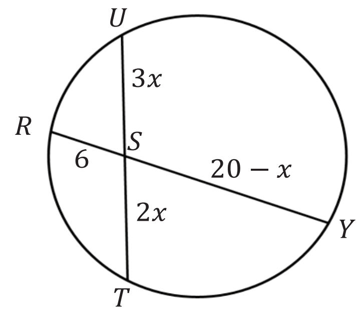
2. Consider the circle below.



Part A: Determine $m\overline{OE}$.

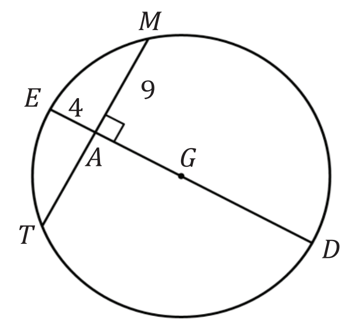
Part B: Determine $m\overline{MY}$.

3. Determine the length of $\overline{UT}$.

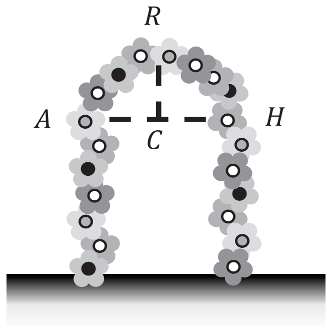


4. Find the radius of circle G below.

Ⓐ 12.1
Ⓑ 20.3
Ⓒ 16.3
Ⓓ 24.3

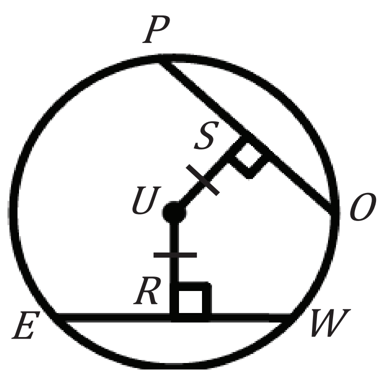


5. The prom is in need of a floral archway, such as the one below. Segment $\overline{RC}$ is the perpendicular bisector of segment $\overline{AH}$. If $AH = 6$ and $RC = 2$, then determine the diameter of the circle that contains $\widehat{AH}$.



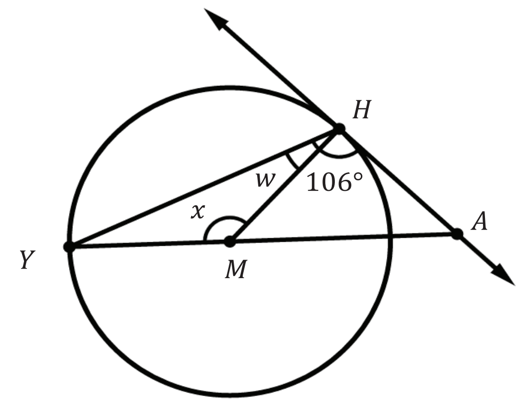
6. Given that $\overline{PO}$ and $\overline{EW}$ are equidistant from the center, determine the value of m and n in the circle below.

$US = 3m + 2$
$RU = -m + 9$
$PO = 20n - 14$
$EW = -2(2n + 1)$

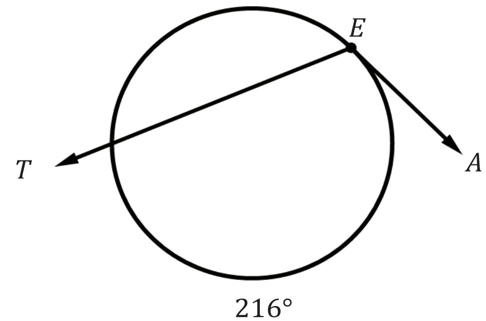


Section 10 – Topic 7
Circumscribed Angles and Beyond – Part 1

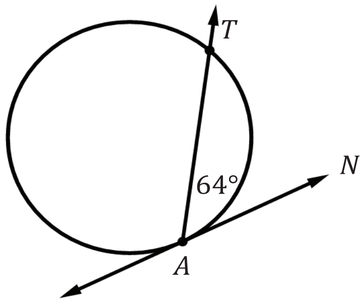
1. Determine the value of w and x in circle M below.



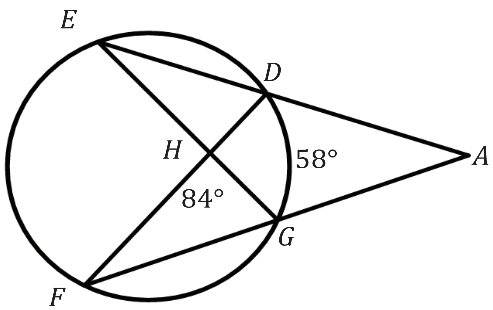
2. Determine $m\angle TEA$.



3. Determine $m\widehat{TA}$.



4. Consider the diagram below.

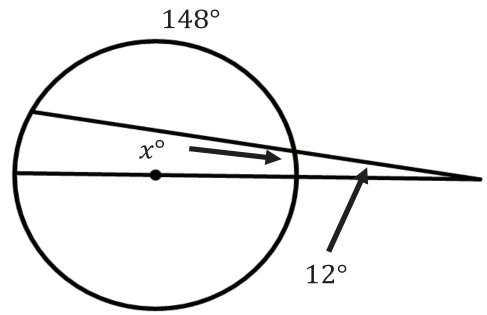


Part A: Determine $m\angle DHG$.

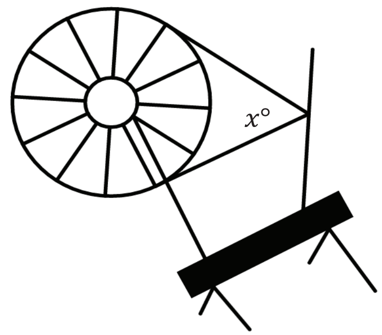
Part B: Determine $m\widehat{EF}$.

Part C: Determine $m\angle EAF$.

5. Determine the value of x in the circle below.

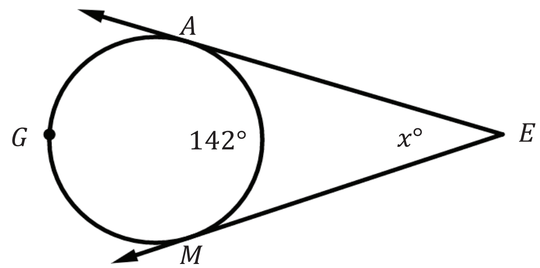


6. A spinning wheel is a device for creating thread or yarn from natural materials. A spinning wheel is a large, usually wooden, circle that is connected to a foot trundle and contains a belt connected to a small bobbin that turns very quickly. If each spoke intercepts a 30° arc, then determine the value of x.

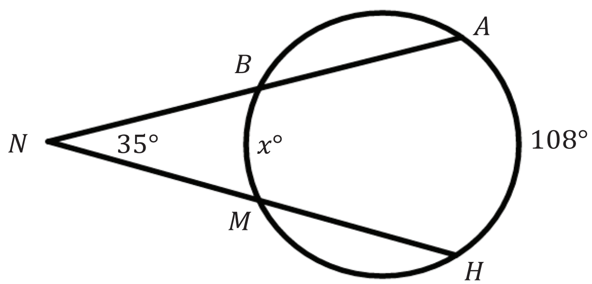


Section 10 – Topic 8
Circumscribed Angles and Beyond – Part 2

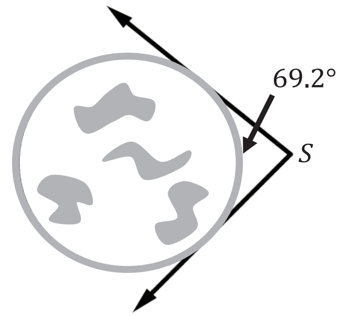
1. Determine the value of x and major arc $\widehat{AGM}$.



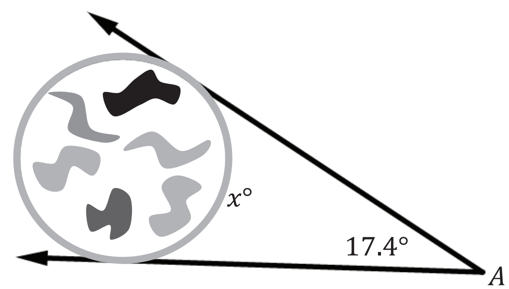
2. Determine the value of x.



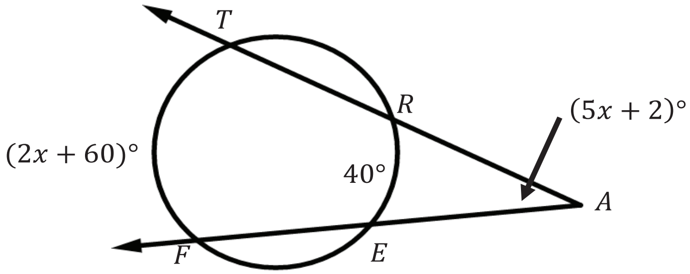
3. A lunar satellite is orbiting approximately 900 kilometers above the surface of the moon. Approximately 69.2 arc degrees of the moon is visible to the camera of the satellite. Determine $m\angle S$.



4. New Horizons is an interplanetary space probe that approached the dwarf planet Pluto in 2015. How many arc degrees of Pluto are visible to the New Horizons space probe while approaching the dwarf planet?

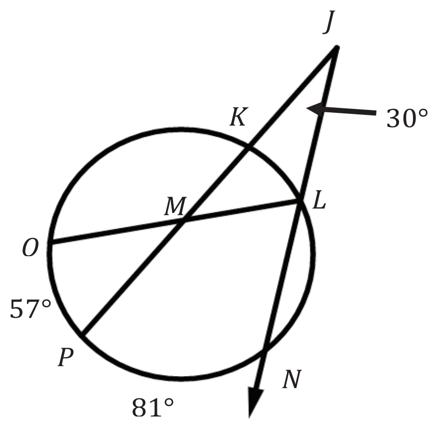


5. Consider the circle below.



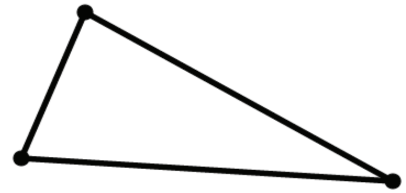
Determine the value of x.

6. Determine the value of $\angle OMP$.

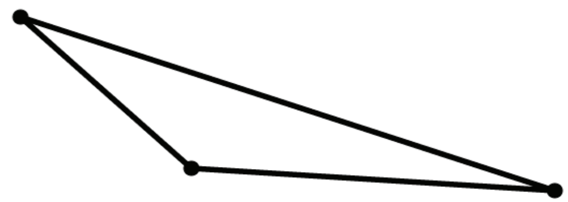


Section 10 – Topic 9
Constructing Inscribed and Circumscribed Circles of Triangles

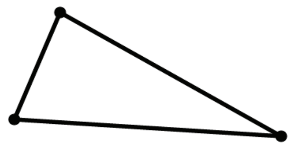
1. Construct an inscribed circle in the acute triangle below.



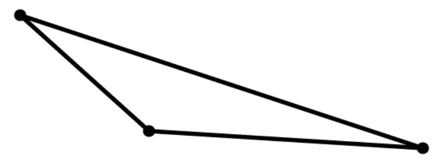
2. Construct an inscribed circle in the obtuse triangle below.



3. Construct a circumscribed circle in the acute triangle below.



4. Construct a circumscribed circle in the obtuse triangle below.



Section 11: Three-Dimensional Geometry
Student Learning Plan

Topic Number	Topic Name	Date Completed	Study Expert(s)	Check Your Understanding Score
1	Geometry Nets and Three-Dimensional Figures			
2	Cavalieri's Principle for Area			
3	Cavalieri's Principle for Volume			
4	Volume of Prisms and Cylinders			
5	Surface Area of Prisms and Cylinders			
6	Volume of Pyramids and Cones			
7	Surface Area of Pyramids and Cones			
8	Spheres			
9	Area in Real-World Contexts			
10	Geometric Design			
11	Volume in Real-World Contexts			
12	Density			
13	Similar Shapes			
14	Cross Sections and Plane Rotations			
Honors 1	Deriving the Equation of a Parabola Given a Focus and Directrix			
Honors 2	Deriving the Ellipse Formula			
Honors 3	Deriving the Equation of a Hyperbola			

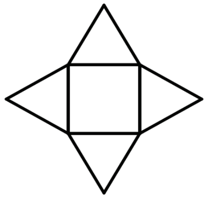
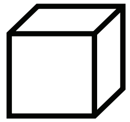
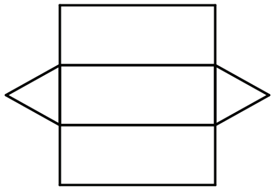
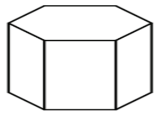
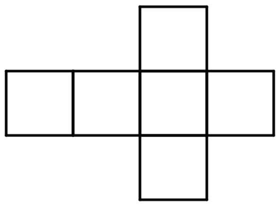
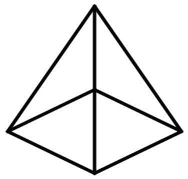
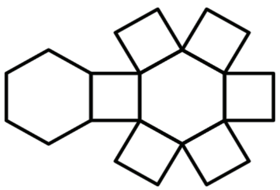
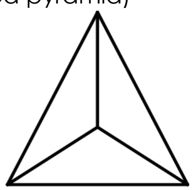
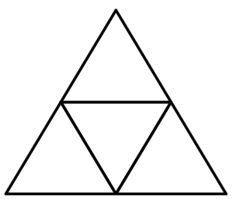
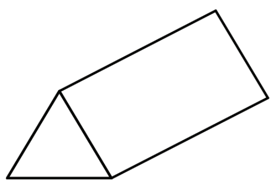
**Honors resources are available online.*

What did you learn in this section? What questions do you still have?

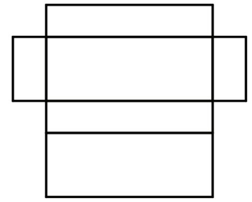
Who was your favorite Study Expert for this section? Why?

Section 11 – Topic 1
Geometry Nets and Three-Dimensional Figures

1. Match the net with the corresponding solid.

_____		**A.** Cube 
_____		**B.** Hexagonal Prism 
_____		**C.** Square Based Pyramid 
_____		**D.** Tetrahedron (triangular based pyramid) 
_____		**E.** Triangular Prism 

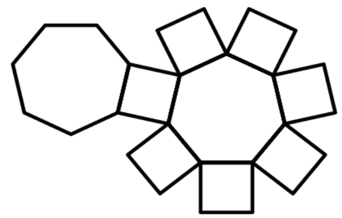
2. Consider the net of a prism below.



Part A: Describe the prism that the net models.

Part B: Draw the prism that the net models.

3. Consider the net below.



Part A: Determine the number of faces and bases in the prism above.

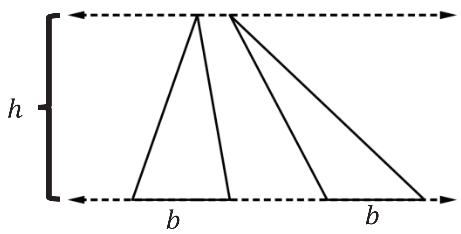
Part B: Describe the height of the prism.

Part C: Describe the surface area of the prism.

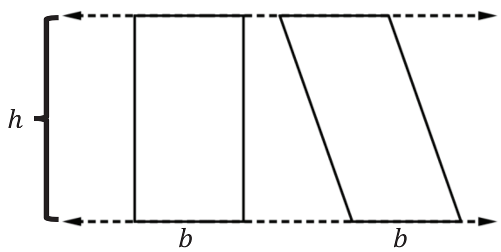
Section 11- Topic 2
Cavalieri's Principle for Area

1. Determine if the following shapes have the same area or not. Justify your answer.

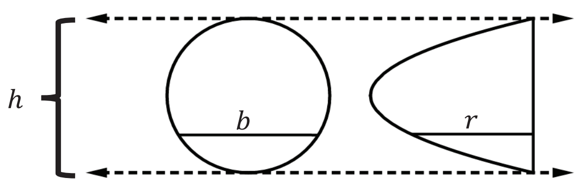
Part A:



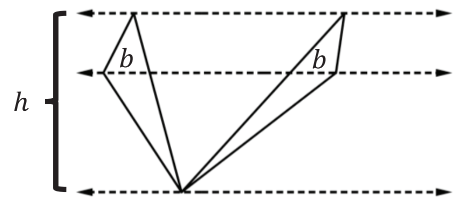
Part B:



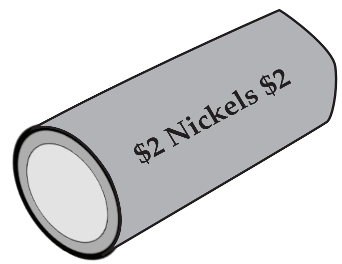
Part C:



Part D:

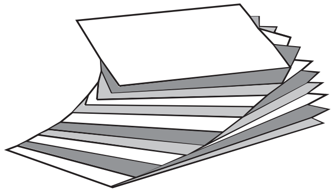


2. Consider the roll of nickels below.



If the diameter of a nickel is $0.835\ in.$ and the width of each nickel is $0.077\ in.$, what is the approximate surface area of the roll of nickels? Round your answer to the nearest hundredth.

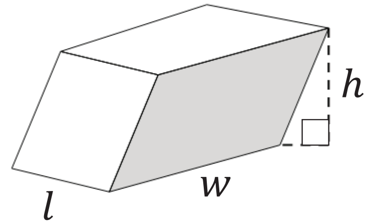
3. Consider the pile of plywood below.



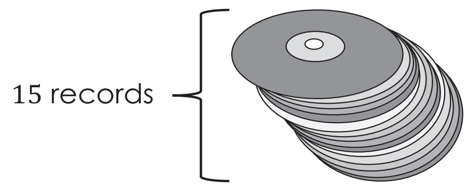
One piece of leftover plywood is 0.75 inches thick, 3 feet wide, and 7 feet long. Estimate the approximate surface area of two stacks of 11-pieces of leftover plywood. Round your answer to the nearest hundredth.

Section 11 – Topic 3
Cavalieri's Principle for Volume

1. Determine the volume for the "tilted prism" if the dimensions are $l = 43$ inches, $w = 92$ inches, and $h = 4$ feet.



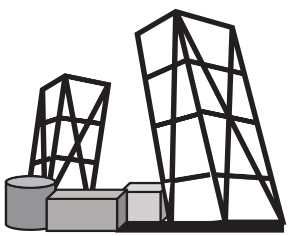
2. Columbia Records unveiled the LP (a vinyl record) in the Waldorf Astoria on June 18, 1948, in two formats: 10 inches in diameter, matching that of 78 rpm singles, and 12 inches in diameter. If the thickness of one vinyl record is $0.112\ in$, then determine the difference in volumes between the 10 inch and 12 inch records.



3. A triangular prism has an isosceles right triangular base with a hypotenuse of $\sqrt{32}$ and a prism height of 12. A square prism has a height of 12 and its volume is equal to that of the rectangular prism. What are the dimensions of the square's base, in simplest radical form?

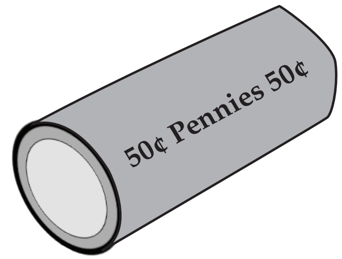
4. Find the volume of an oblique circular cylinder that has a radius of five feet and a height of three feet.

5. The Gate of Europe towers are rectangular prisms, known as KIO Towers, and twin office buildings in Madrid, Spain. The towers have a height of 114 meters or 374 feet. They are the first inclined skyscrapers with a tilt of $15°$ off perpendicular. If their bases are 205 feet by 180 feet, then determine volume inside both towers.



Section 11 – Topic 4
Volume of Prisms and Cylinders

1. Consider the roll of pennies below.



The diameter of a penny is $19.05\ mm.$ and the thickness is $1.52\ mm.$ What is the approximate volume of the roll of pennies, to the nearest tenth?

2. Consider a prism with a height of 8 inches and a 14 –inch by 14 –inch square base. Consider a cylinder with a height of 10 inches and a diameter of 10 inches. Determine the difference in volume between the two solids.

3. Créstòr bought a terrarium that is 24 inches wide, 48 inches long and 20 inches high. A bag of special dirt at the pet store fills only $1{,}000$ cubic inches and costs $\$8.00$ per bag.

   Part A: How many bags does Créstòr need to fill his terrarium?

   Part B: How much do the bags cost after a 6% sales tax?

4. A large "round" bale of hay is actually in the shape of a cylinder with a four–foot diameter and a five–foot height. A traditional "square" bale of hay is actually in the shape of a rectangular prism. Its dimensions are two feet by four feet by two feet. Approximately how many "square" bales contain the same amount of hay as one "round" bale of hay?

5. A tank on the road roller is filled with water to make the roller heavy. The tank is a cylinder that has a height of six feet and a radius of two feet. Since one cubic foot of water weighs 62.5 pounds, determine the weight of the water in a full tank.

6. The store salsa jars have a height of ten centimeters and a radius of five centimeters. If there is only four centimeters of salsa left in the jar, then determine how much salsa is missing from the jar.

7. An office water cooler has a height of 1.7 feet and a base diameter of one foot. About how many gallons of water does the water cooler bottle contain if $1\ ft^3 = 7.5$ gallons.

   Ⓐ 5.3 gallons
   Ⓑ 10 gallons
   Ⓒ 17 gallons
   Ⓓ 40 gallons

Section 11 – Topic 5
Surface Area of Prisms and Cylinders

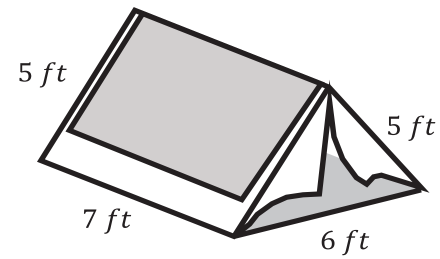
1. The average bedroom in an apartment is approximately $16' x 10'$ with an eight-foot ceiling. As an interior decorator that loves the $1970's$ décor, you need to cover the walls with groovy flowered wallpaper.

   Part A: If one roll of wallpaper covers $50\ ft^2$, then determine how many rolls of groovy flowered wallpaper is needed to cover the apartment bedroom.

   Part B: If one roll of paper cost $\$15.90$, then determine the cost to re-wallpaper the apartment room.

2. A fuel-tanker's tank is 50 feet long and has a diameter of seven feet. Determine the surface area of the fuel tank.

3. Consider the ready-made tent with flooring included that is shown in the diagram below.



   What is the least amount of fabric needed to make the four-foot high tent?

4. The material used to make a storage box costs $\$1.25$ per square foot. The boxes have the same volume, so how much does a company save by choosing to make 50 of Box 2 instead of Box 1?

	Length	Width	Height
Box 1	$20\ in$	$6\ in$	$4\ in$
Box 2	$15\ in$	$4\ in$	$8\ in$

5. A can of sweet peas at the local grocery store has a radius of one inch and a height of two inches. How much paper is used for the label on the can of peas?

6. Your Humanities class is having a recycling project where you have to collect tin cans. If you earn $\$0.02$ for recycling the sweet peas can from Question 5, then how much can you expect to earn for recycling a tomato can that has a two-inch radius and a height of 5.5 inches? The recycle value is proportional to the surface area.

Section 11 – Topic 6
Volume of Pyramids and Cones

1. The Cholula Pyramid in Mexico has a height approximately 217 feet and a base approximately 1476 feet by 1476 feet. The Cheops Pyramid (part of the Great Pyramids) is taller with an approximate height of 480 feet but with a 755 foot by 755 foot base. Determine which pyramid has the most volume.

2. In 1483, Leonardo da Vinci designed, possibly, the first parachute in the shape of a pyramid with a square having a 12-yard base that is 12 yards high. Determine the volume of air inside Leonardo's parachute.

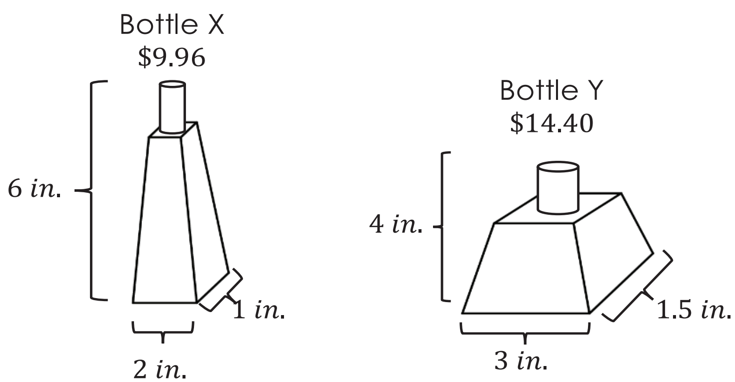
3. Monica sells lemonade over the summer and uses conical paper cups to serve the lemonade. Each cup is 8 centimeters in diameter and 11 centimeters tall. She starts with a 10-gallon container of lemonade (one gallon is equivalent to $3785\ cm^3$).

   Part A: Determine how many paper cups are needed to empty the entire 10-gallon container of lemonade.

Part B: If the conical cups are sold in packages of 50. How many packages does she need?

4. Determine the height of a cone that has a diameter of 10 inches and a volume of $225\ in^3$.

5. At *Shower and Body Works*, there are different sizes of lotion dispensers. Two of the versions are shown below.



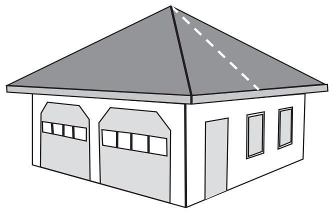
Part A: How many times more lotion is in Bottle Y than in Bottle X?

Part B: Which bottle is the better buy?

Part C: If a third bottle on sale for $\$13.20$ has a height of three inches and a base area of six square inches, then determine which bottle is a better buy, Bottle Y or the third bottle. Justify your answer.

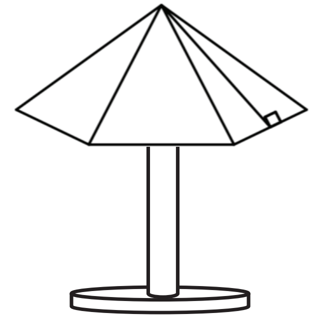
Section 11 – Topic 7
Surface Area of Pyramids and Cones

1. The roof of the house down the road is in the shape of a square pyramid. The length from the gutter to the top is 15 feet and the length of gutter on one side is 18 feet with no overlap. One bundle of roof shingles covers 25 square feet. How many bundles should you buy to cover the roof?



2. If the area of a pentagonal pyramid has a $440.4\ mm^2$ base area, a $16\ mm$ pentagon side length, and a $20\ mm$ slant height, then determine the surface area of the pyramid.

3. The base of the lampshade is a regular hexagon with a side length of eight inches and a slant height of 10 inches. Estimate the amount of glass needed to make the lampshade

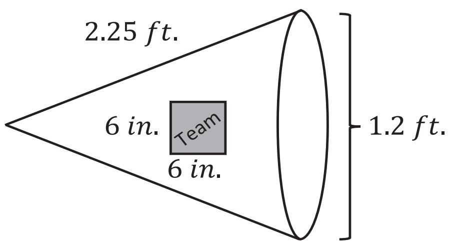


4. The surface area of a square pyramid is 85 square meters. The base length is five meters. Determine the slant height.

5. An umbrella you bought is shaped like a regular octagonal pyramid with a side length of four feet and a slant height of five feet. Estimate the amount of fabric in the umbrella.

6. A roof on a castle tower is shaped like a cone with a diameter of 12 feet and has a slant height of 13 feet. One bundle of roof shingles covers 32 square feet. How many bundles should you buy to cover the roof?

7. *MegaPhone Design Inc.* designs megaphones with sport team stickers on opposite sides of the megaphone. Estimate the percent of the surface area of the megaphone covered by the stickers. Round to the nearest percent.



8. Find the surface area of the cone with a diameter of 12 centimeters and a slant height of 85 millimeters.

Section 11 – Topic 8
Spheres

1. Consider a sphere with a radius with an 18 inches.

   Part A: Determine the surface area of the sphere.

   Part B: Determine the volume of the sphere.

2. A sphere has a diameter of $4(x+3)$ centimeters and a surface area of 784π square centimeters. Find the value of x.

3. *DC SPORTS Inc* ships sports balls around the world. A bowling ball has a diameter of 8.5 inches. A basketball has a circumference of 29.5 inches. A softball has a circumference of 12 inches. A golf ball has a diameter of 1.7 inches.

   Determine which package of balls has the most volume: One bowling ball, one basketball, four dozen golf balls, or 50 softballs. Justify your answer.

4. The Torrid Zone on Earth is the area between the Tropic of Cancer and the Tropic of Capricorn (lines of latitude close to the equator). The distance between these two tropics is about 3250 miles. You can estimate the distance as the height of a cylindrical belt around the earth at the equator.

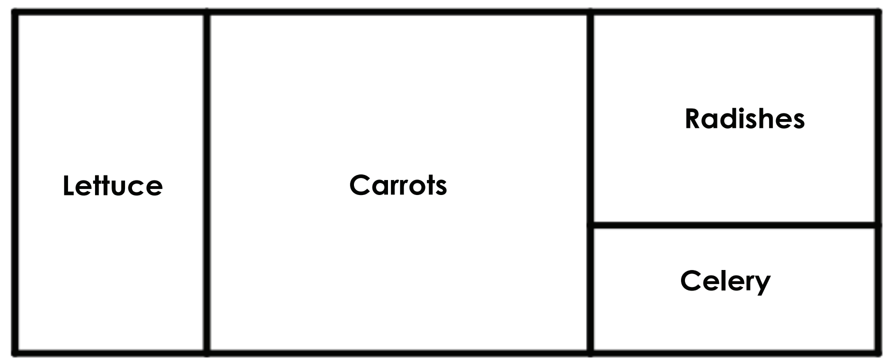
   Part A: If the radius of the Earth is 3960 miles, then estimate the amount of surface area of the Torrid Zone. Approximate to the nearest thousands place.

   Part B: A meteorite is equally likely to hit the earth. Estimate the probability of a meteorite hitting the Torrid Zone.

Section 11 – Topic 9
Area in Real-World Contexts

1. Carissa is helping her grandfather plan his vegetable garden. Carissa's grandfather has given her some information about each of the sections. Carissa drew a model of the garden showing how the garden will be divided. The entire length of the garden is 62 feet and the combined area of the zone where the radishes and the celery are located is 528 square feet. Additional information is provided below.

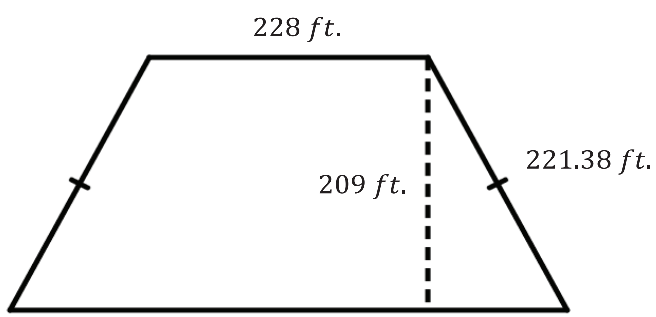
Radishes	Celery	Carrots	Lettuce
- Rectangle	- Rectangle - Perimeter = $64\ ft$	- Square - Perimeter = $96\ ft$	- Rectangle



Determine the dimensions of the plots for each crop.

2. The area of Miguel's rectangular garden is 450 square feet. The garden is 9 feet wide. How many feet of fencing will Miguel need to buy to enclose the garden on all four sides?

3. A recent tornado in a small town in southwest New York destroyed and damaged multiple plots of land. The insurance company has been surveying these plots of land to write their report on the damage assessment. One plot of land is the shape of an isosceles trapezoid, and the insurance must determine the area of the land.



Determine the area of the plot of land above.

Section 11 – Topic 10
Geometric Design

1. Kathrine, Giana, and Jackson are designing jewelry boxes to sell at the art fair. The dimensions of their boxes are:

   Kathrine: $5in \times 5in \times 5in$
   Giana: $10in \times 12in \times 2in$
   Jackson: $10in \times 5in \times 4in$

   If a customer at the art fair is wants to buy the jewelry box that will hold the most amount of jewelry, whose jewelry box should they purchase?

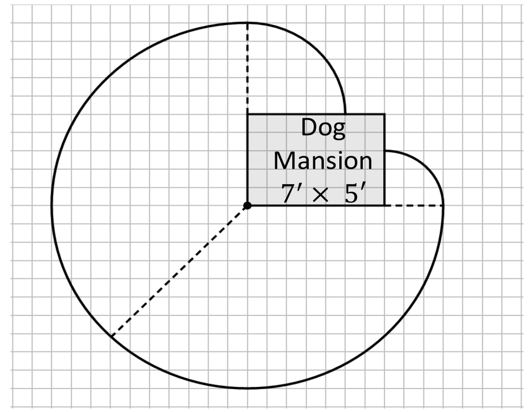
2. Patricia is building the community dog park. She plans to build the dog park right beside the city park so she can use one side of the existing fence. Her budget allows her to purchase 340 feet of fencing. In order to make the area of the dog park as large as possible, determine the dimensions of the dog park if one side of the fence is attached to the city park's fence.

3. Two rectangular corrals are to be made from 100 yds. of fencing. If a rancher wants the total area to be maximized, what should the dimensions of the corral be?

4. A major convention center, whose floors are covered by square carpet tiles with side lengths of 9 inches laid out in a 48 by 96 array, held a wild conference over the weekend. A local cleaning company was hired to clean the carpet tiles, given an offer of $\$0.05$ per carpet square. The cleaners accepted the job, but not until after the convention center agreed to their counter of $\$0.10$ per square foot. The cleaners were happy, because they knew they would be getting paid more than the convention center originally offered. How much more, exactly?

Ⓐ $\$11.52$
Ⓑ $\$28.80$
Ⓒ $\$115.20$
Ⓓ $\$201.60$

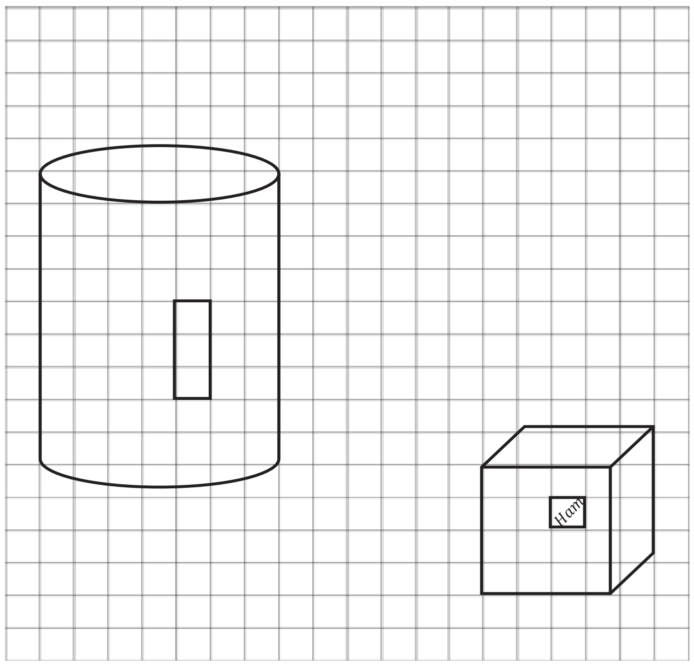
5. Fido's domain is limited by a 10 –foot long leash that is tethered to the corner of his expansive 7 by 5 –foot dog mansion. How much outdoor territory can Fido mark?



Ⓐ 262.19 square feet
Ⓑ 235.5 square feet
Ⓒ 314 square feet
Ⓓ 297.19 square fee

6. At a food packaging company, workers pack soda crackers into big cans and ham slices into boxes. Use the diagram below to estimate the amount of soda crackers workers can fit into a can and the amount of ham slices workers can fit into a box.

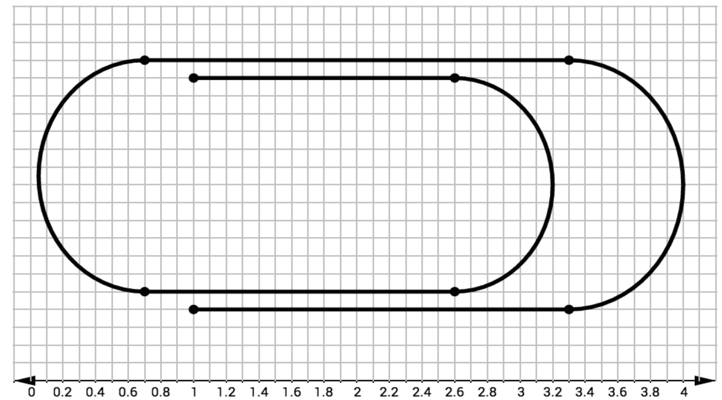
   Each grid on the graph below represents a square inch. Each soda cracker is an eighth of an inch thick and each ham slice is a fourth of an inch thick. The crackers and ham are stacked on top of each other in their respective containers.



Workers can pack [ ] ham slices into a box

Workers can pack [ ] soda crackers into a can.

7. Consider the paper clip below that is approximately four centimeters long. As a team, Donna and Jon need to determine the total length of wire one paperclip requires. Donna plots inflection points on the paperclip that separate linear portions and non-linear portions (which are exact semi-circles). Each grid line measures 0.1 of a centimeter.



Part A: Determine the approximate length of linear sections of the paperclip.

Part B: Determine the approximate length of the semi-circle sections of the paper clip.

Part C: Determine the total length of wire needed to make one paper clip.

Part D: If the company that makes paperclips receives the wire in 10 meter rods, then determine the number of paperclips one $10-$meter rod can produce.

Section 11 – Topic 11
Volume in Real-World Contexts

1. A pedestal on which a statue is raised on is a rectangular concrete solid base measuring nine feet long, nine feet wide, and six inches high. How much is the cost of the concrete in the pedestal, if concrete costs $**70** per cubic yard?

2. Mr. Gomez stores his iguana food in a can that is eight inches tall and has a diameter of six inches. He stores his hamster food in a can that is ten inches tall and has a diameter of five inches. Which can of food is larger?

3. Zugo delivers muffins for the Pastry-and-Muffin company. Each muffin is packed in its own box. An individual muffin box has the shape of a cube, measuring three inches on each side. Zugo packs the individual muffin boxes into a larger box. The larger box is also in the shape of a cube, measuring two feet on each side. How many of the individual muffin boxes can fit into the larger box?

4. Euclid's root beer mug is shaped like a cylinder that is eight inches tall with a radius of three inches. Aristotle's root beer glass is shaped like a cone that is 18 inches tall with a diameter of four inches. Which mug holds the most root beer?

5. Plato stores his Pokémon cards in a shoe box measuring eight inches by 14 inches by six inches. Socrates stores his Magic cards in a cake box measuring one foot by one foot by five inches. Whose box has the greater capacity?

Section 11 – Topic 12
Density

1. A student measures the mass of an $8\ cm^3$ block of confectioner's powdered sugar to be 4.49 grams. Determine the density of the powdered sugar.

2. Determine the weight of ethyl alcohol that exactly fills a $300\ mL$ container if the density of the ethyl alcohol is $0.789\ g/ML$.

3. If a block of lead has dimensions of a 5.2 inches by 4.5 inches by 6.0 inches, is a rectangular prism and weighs 1587 grams, then determine the density of the block of lead.

4. The cool science teacher performs a demonstration and finds that a piece of cork displaces $23.5\ mL$ of water and weighs 5.7 grams. What is the density of the cork?

5. An empty graduated cylinder has a mass of 50 grams. The cylinder has a mass of 120 grams when 30 milliliters of water is poured into it. When a rock is added to the graduated cylinder, the water level rises to 75 milliliters and the mass is 250 grams. Determine the density of the rock.

6. Consider the table below of different types of solids and their densities.

Solid	Density g/cm^3
Copper	8.92
Gold	19.32
Platinum	21.4
Marble	2.56
Quartz	2.64
Diamond	3.52

Part A: While digging in the backyard, you find an old coin. If the coin's mass is 26.7 grams and the volume is 3 centimeters, determine what the coin is made of.

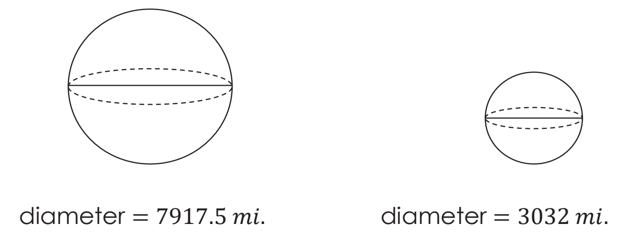
Part B: A rectangular block of material that acts as a paperweight is on your dad's desk. The measurements are $3\ cm$ x $4\ cm.$ x $6\ cm.$ and it has a mass of 184.32 grams. Determine what the paperweight is made of.

Part C: In the play-pit of a restaurant, you find a ring with a mass of 107 grams. You use water displacement to determine that the ring's volume is $5\ mL$. Determine what the ring is made of.

Part D: Eureka! You think you found a diamond! The mass is 5.28 grams and the volume is 2 cubic centimeters. Determine the type of solid you found and if you hit the jackpot.

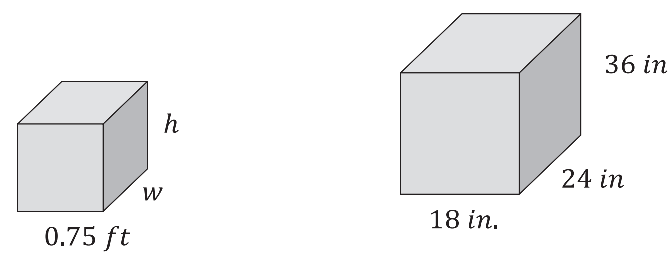
Section 11 – Topic 13
Similar Shapes

1. Consider the diagram below of the Earth and the planet Mercury.



Determine how many times Mercury can fit "inside" the Earth.

2. Consider the figures below that represent two similar solids. Find the value of the missing dimensions and support your answer.



3. Two cylinders, A and B, are mathematically similar. The height of B is twice the corresponding height of A. The volume of A is $13\ cm^3$. Find the volume of B.

4. Two rectangular prisms, M and N, are mathematically similar. The volumes of M and N are $17\ cm^3$ and $136\ cm^3$, respectively. The height of N is $18\ cm$. Find the corresponding height of M.

5. A MP8 Mack Semi Truck Diesel Engine has the volume of 780 cubic inches.

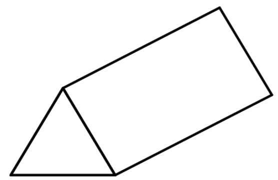
   Part A: Which scale model has the greatest engine volume, a $1:8$ scale model or a $1/24$ scale model?

   Part B: How much greater volume does the bigger scale model have?

6. A small water silo has a diameter of 30 feet and a height of 60 feet. The clay model the engineers used has a height of only 10 inches with proportional volume. Determine the diameter of the model.

Section 11 – Topic 14
Cross Sections and Plane Rotations

1. *Shutterfly*, an online picture printing company, ships triangular prisms for pictures greater than $8x10$ inch in dimensions.



Which of the following images models a bird's eye view of the top of the Shutterfly box?

Ⓐ 

Ⓑ 

Ⓒ 

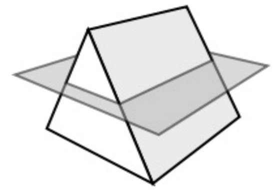
Ⓓ 

2. For a right square pyramid, which shape could not be a cross section?

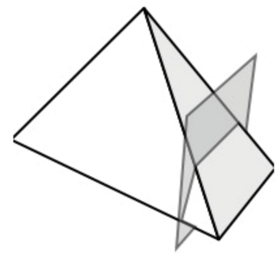
Ⓐ Square
Ⓑ Triangle
Ⓒ Trapezoid
Ⓓ Rectangle

3. Determine the shape of the cross section that would be created if the 3D shape were sliced as shown.

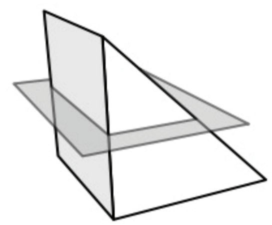
Part A:



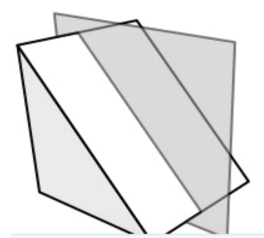
Part B:



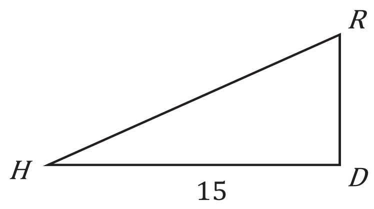
Part C:



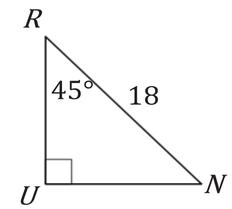
Part D:



4. What is the surface area of the geometric solid produced by the $30-60-90$ triangle below when it is rotated 360 degrees about the axis RD? Support your answer.



5. What is the volume of the geometric solid produced by the triangle below when it is rotated 360 degrees about the axis RU? Support your answer.



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